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Arithmetical Abilities

and

Some Factors Determining Them

BY

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No school system will appear by name in the body of this study, but any one whose system is represented will be cordially provided with its key, of which three copies will be preserved—one by the author and two at Teachers College, one in the Office of Elementary Education and one in the Bryson Library.

Dr. Rice's study of arithmetic and other helpful works are noted and briefly summarized in the appendix, page 96, or throughout the study in connection with related topics.

PART I.—PURPOSE AND METHOD

The central *purpose* of this study, broadly stated, is to make one more contribution to exact knowledge of the relation between distinctive educational procedures and the resulting products. "What is the relation between the theories and the products of education?" is one statement of the question which prompted the author to attempt a research thesis. Specifically, the portions of the larger question on which this study has bearing are: (1) What is the nature of the product of the first six years of arithmetic work? (2) What is the relation between distinctive procedures in arithmetic work and the resulting abilities?

The *method* of this inquiry may be briefly characterized as the application of the statistical method to mental measurements.[1] The three principal measurements made are the arithmetical abilities of 6A (high 6th) pupils,[2] the time expended and the course of study materials used in securing these abilities.

Sources of Data

The sources drawn upon for data with which to answer the questions above stated were twenty-six school systems. These were selected after a somewhat close examination of the arithmetic courses of study and time allotments of a large number of city and individual schools scattered over practically the entire United States. The bases of selection were distinctiveness of practice and geographical accessibility.

Through the courtesy of superintendents, principals and teachers the data were gathered in the following twenty-six systems:[3]

Batavia, New York.
Decatur, Illinois.
Elwood, Indiana.

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[1]For a full exposition, see Thorndike's *Mental and Social Measurements*.
[2]In such of the smaller systems as had less than one hundred 6 A's, tests were used from equal numbers of 6 B's and 7 B's.
[3]The author personally gathered the data from each system, securing course of study material from each of the twenty-six superintendents, and time expenditure data from each of seventy-five principals, and conducting the tests in each of the one hundred and fifty-two classes. See p. 13, Conditions under which the Tests were given. The single exception is the Ethical Culture School where, because of the author being known to the children, Mr. J. L. Stockton, to whom the thanks of the author are due, gave the tests after careful preparation.

Ethical Culture School, New York City.
Francis W. Parker School, Chicago.
Horace Mann School, Columbia University, New York City.
Indianapolis, Indiana.
Jersey City, New Jersey.
Kokomo, Indiana.
Linné School, Chicago.
Medford, Massachusetts.
Montclair, New Jersey.
Muncie, Indiana.
Natick, Massachusetts.
Observation School, State Normal, Providence, R. I.
Passaic, New Jersey.
Providence, Rhode Island.
Rochester, New York.
Schools No. 40 and No. 50, Manhattan, New York City.
Speyer School, Columbia University.
Syracuse, New York.
Training School, State Normal, Hyannis, Massachusetts.
University Elementary School, University of Chicago.
Waltham, Massachusetts.
Waukegan, Illinois.
Yonkers, New York.

It will be noted that of these twenty-six systems there are seventeen city public systems—Batavia, Decatur, Elwood, Indianapolis, Jersey City, Kokomo, Medford, Montclair, Muncie, Natick, Passaic, Providence, Rochester, Syracuse, Waltham, Waukegan and Yonkers; three schools in city systems—Linné, School No. 40 and School No. 50; four training schools—Observation School, Speyer School, Hyannis Training School and University Elementary School; and three private schools—Ethical Culture, Francis W. Parker and Horace Mann.

According to geographical location these schools may be grouped as follows:

Six in New England, namely Medford, Natick, Observation School, Providence, Training School and Waltham; eleven in the Middle East, namely Batavia, Ethical Culture, Horace Mann, Jersey City, Montclair, Passaic, Rochester, Schools No. 40 and No. 50, Speyer School, Syracuse and Yonkers; and nine in the

Middle West, namely Decatur, Elwood, Francis W. Parker, Indianapolis, Kokomo, Linné School, Muncie, University Elementary School and Waukegan.

Readers who know these schools will recognize that they represent a diversity of practice as to the relative reliance placed on the various factors supposed to produce arithmetical abilities. There are among these systems distinctive procedures in regard to at least the following—*the time devoted to arithmetic and its distribution among grades; the supervision of the teaching of arithmetic; and the nature and arrangement of the subject matter taught.*

As stated under acknowledgments, no system will appear by name in connection with its relative standing, each system having been assigned a Roman numeral in a key. The number of each respective system bears no relation whatever to any rating of that system, the numbers having been determined by a chance distribution of the twenty-six systems among the first twenty-six Roman numerals.

Method of Securing Data

In one word the method of securing data may be characterized as *personal.* Tests to measure arithmetical abilities were personally conducted by the author; blanks calling for the time expenditure and other information were filled out by the principals and teachers at the author's personal request during the time the tests were being given; and after an interview with the author, the courses of study were rated by twenty-one unbiased judges each of whom had had practical experience with the workings of various courses of study and had acquired a knowledge of the principles underlying their construction.

Purpose and Content of the Tests

In Fundamentals[1]

The main purpose of the test in fundamental operations was the determination of the ability of Grade VI A (high 6th) pupils

[1]The word *fundamental* and the words *fundamental operations* are used throughout this study to designate the more formal aspects of arithmetic work, viz., addition, subtraction, multiplication and division of whole numbers. It will be recognized that in the sense of priority of time these so-called fundamentals are not more fundamental than certain aspects of reasoning.

in addition, subtraction, multiplication and division. To this end the test was meant to embody all the difficulties of the four fundamental operations in sufficient quantities to thoroughly test any VI A Grade. The test is purposely too long for any pupil to finish entirely in the twelve minute limit.

In Reasoning

The main purpose of the reasoning test is the determination of the ability of VI A children to reason in arithmetic. To this end, the problems as selected and arranged, are meant to embody the following conditions:

1. Situations equally concrete to all VI A children.

2. Graduated difficulties.
   a. As to arithmetical thinking.
   b. As to familiarity with the situation presented.

3. The omission of
   a. Large numbers.
   b. Particular memory requirement.
   c. Catch problems.
   d. All subject matter except whole numbers, fractions, and United States money.

The test is purposely so long that only very rarely did any pupil fully complete it in the fifteen minute limit.

The Tests as Given

The following are reproductions of the tests. They were printed separately and each pupil was furnished with a copy.

Work as many of these problems as you have time for; work them in order as numbered.

1. Add

```
2375
4052
6354
 260
5041
1543
----
```

2. Multiply 3265 by 20.

3. Divide 3328 by 64.

4. Add

596
428
94
75
302
645
984
897

5. Multiply 768 by 604.

6. Divide 1918962 by 543.

7. Add

4695
872
7948
6786
567
858
9447
7499

8. Multiply 976 by 87.

9. Divide 2782542 by 679.

10. Multiply 5489 by 9876.

11. Divide 5099941 by 749.

12. Multiply 876 by 79.

13. Divide 62693256 by 859.

14. Multiply 96879 by 896.

Solve as many of the following problems as you have time for; work them in order as numbered:

1. If you buy 2 tablets at 7 cents each and a book for 65 cents, how much change should you receive from a two-dollar bill?

2. John sold 4 Saturday Evening Posts at 5 cents each. He kept ½ the money and with the other ½ he bought Sunday papers at 2 cents each. How many did he buy?

3. If James had 4 times as much money as George, he would have $16. How much money has George?

4. How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?

5. The uniforms for a baseball nine cost \$2.50 each. The shoes cost \$2 a pair. What was the total cost of uniforms and shoes for the nine?

6. In the schools of a certain city there are 2,200 pupils; ½ are in the primary grades, ¼ in the grammar grades, ⅛ in the High School and the rest in the night school. How many pupils are there in the night school?

7. If 3⅓ tons of coal cost \$21, what will 5½ tons cost?

8. A news dealer bought some magazines for \$1. He sold them for \$1.20, gaining 5 cents on each magazine. How many magazines were there?

9. A girl spent ⅛ of her money for car fare, and three times as much for clothes. Half of what she had left was 80 cents. How much money did she have at first?

10. Two girls receive \$2.10 for making button-holes. One makes 42, the other 28. How shall they divide the money?

11. Mr. Brown paid one-third of the cost of a building; Mr. Johnson paid ½ the cost. Mr. Johnson received \$500 more annual rent than Mr. Brown. How much did each receive?

12. A freight train left Albany for New York at 6 o'clock. An express left on the same track at 8 o'clock. It went at the rate of 40 miles an hour. At what time of day will it overtake the freight train if the freight train stops after it has gone 56 miles?

The Arrangement of Problems and the Determination of the Time Limits.

In Fundamentals

The test in fundamental operations was so arranged as to enable the pupils to meet the main difficulties in the first six problems. These were meant to increase in difficulty as were the next three; and problems ten to fourteen were included so as to furnish enough work for the most rapid pupils. After several preliminary trials, the time limit was set at twelve minutes as the time best suited to allow the average of the slower classes to encounter the main difficulties and to keep all of the most rapid pupils from getting out of work.

In Reasoning

The arrangement of the problems in reasoning was determined by a preliminary test. After embodying the desired con-

tent (see page 10) the problems were given in chance order to one hundred representative high 6th pupils, to determine their relative difficulty. As the pupils were given as much time as they needed in order to work each problem as well as they could, the per cent of problems worked showed their relative difficulty. On this basis the problems were arranged with the less difficult first and printed as seen on page 11. After several preliminary trials the time limit was set at fifteen minutes as the time in which a majority of pupils worked through the first six or seven problems and in which practically no pupil completed the test.

Conditions under Which the Tests Were Given

I. Personally Conducted by the Author.
1. All directions given orally by the author, except as printed on test slips. See page 14 for directions to pupils.
2. Time limits kept exactly.
Twelve minutes for fundamentals.
Fifteen minutes for reasoning.
3. Each pupil provided with a printed copy of each test.

II. Superintendents were asked:
1. To select the four or five buildings containing high 6th pupils in which the approved course of study had been best carried out for the past five and one-half, or more years.
2. To refrain from announcing the tests to principals or teachers.
3. Not to be in and not to go into the room that was tested.
4. To furnish a note to principals and 6 A (high 6th) teachers, similar to the following:

Mr. Stone is giving the tests in several cities, and, in order to make the work as accurate as possible, he is anxious to have all children follow exactly the same directions under exactly the same conditions. To this end he asks that principals and teachers refrain from making any announcement of the tests to the pupils, and that they have as little as possible to do with the pupils during the tests.

III. Principals were asked:

1. To introduce the author to the teacher—preferably by note.
2. Not to make any announcement to the pupils.
3. Not to be in the room at all, during or immediately previous to the tests.
4. To fill out the outline calling for data during the time of the tests. (A few asked to keep it longer, that they might make it more accurate.)

IV. Teachers were asked:

1. To allow the author to give all the directions to the pupils.
2. To fill out the outline calling for data during the tests.

V. Other conditions observed:

1. All the 6 A's in a building tested.
2. About two or three schools tested in the forenoon and two in the afternoon.[1]
3. The fundamental operation test was given first in all cases, immediately followed by the reasoning test; twelve minutes for fundamental operations, fifteen minutes for reasoning.
4. In case a school selected by the superintendent proved to be abnormal, the results were not used, e. g., a school having a large percentage of Hebrews of foreign parents would not be a fair measure.
5. Nothing was said to the pupils about scratch-paper, working in steps, amount of work put down, etc.
6. No mention was made to pupils of the time limit.

DIRECTIONS TO PUPILS

[As given *orally* to each class]

Fundamental Operations

I. Take the materials that you usually take for an arithmetic test. Prepare two sheets of paper—headings and all. Have two sheets ready in case you may need them. Use pencils. Keep slips with printing turned down until we are ready to begin.

[1]See conclusion of Miss King's Study, p. 14.

II. Now do you have everything? In order for you to do your best in this test, you will need to do just as all the other boys and girls who have taken this test have done. So pay close attention and then do just as I ask you to do.

III. You will not need to mark these (slips) papers at all. You will find directions at the top of these (slips) papers, telling you just what to do, so you will not need to ask any questions, and —(I do not think I need to say this to you, but I will just because I have said it to all the other boys and girls)—be especially careful *not* to see anybody else's work. It is not easy not to see, but if you pay close attention to your own work only, the test will be the best.

IV. Begin.

[Exactly twelve minutes allowed]

Reasoning

I. Have two sheets prepared again. You may not need both but have them ready.

II. Keep the printing turned down until we are ready to begin—the same as before. Now are you all provided?

III. Begin.

[Exactly fifteen minutes allowed]

How the Scores were Computed

In Fundamentals

As stated under *Arrangement of Problems,* page 12 the problems in fundamentals were meant to increase in difficulty. With the purpose of determining their relative difficulty and thus securing a basis for scoring, the test was given to one hundred 6th grade pupils. As the pupils were given all the time they needed to do the problems as well as they could, the per cent doing each correctly would indicate the relative difficulty. The results of this test are shown in table I below.

TABLE I

Preliminary Test

Fundamentals—Unlimited Time

Per cent of 100 Grade VI Pupils Doing Correct Work

Numbers of Problems	Addition				Subtraction			Multiplication				Division			
1..........	95	93	94	96	...	...	...	...	...	...	...	...	...	...	...
2..........	...	...	...	...	...	...	...	...	...	99	100	...	...	...	...
3..........	...	...	...	...	...	...	100	...	...	98	99	...	...	98	99
4..........	...	92	89	96	...	...	...	...	...	...	...	...	...	...	...
5..........	...	...	97	98	...	...	...	...	98	98	100	...	...	...	...
6..........	...	...	...	...	98	97	96	99	94	94	96	97	96	98	98
7..........	93	94	91	92	...	...	...	...	...	...	...	...	...	...	...
8..........	100	98	99	100	...	...	...	...	...	95	94	...	...	...	...
9..........	...	...	...	...	...	99	99	97	90	98	98	93	96	96	94
10..........	...	...	...	...	...	...	...	91	85	86	96	...	...	...	...

This table shows the data from which the scoring of the fundamentals was determined. In this table, as in the entire study, the results were tabulated by steps, e. g., in the addition problems a score was given for each column added correctly; in the multiplication problems a score was given for each partial product and for each column in addition.

It was expected that a less number of children would be able to add the long columns than the short columns correctly; and that scores could be assigned to the respective problems according to difficulty as was done in the reasoning test;[1] but as is seen in table I, in this test, one step of a problem in fundamentals is about equal to another, be the step long or short—e. g., 96% of the children did the first column of problem one correctly, and exactly the same per cent did the longer and presumably harder column one of problem four correctly. Similarly, it is seen by examining table I that about as many pupils failed in the very

[1]See table II, p. 18.

short additions of the partial products in the multiplication problems as failed in the long columns of the addition problems. And table I also shows that the number of mistakes in addition somewhat exceeded those for any one of the other operations.[1] This fact is again brought out in the coefficients of correlation of standings of systems in fundamentals—see table XVI, page 39. Therefore the method of scoring adopted is that of assigning an arbitrary score of *one* to each step of each problem, e. g., each pupil who added the first column of problem one correctly made one score for his school; each pupil who added all four columns of problem one correctly made a score of four for his school; each pupil who did problem five correctly made a score of five for his school, three in multiplication and two in addition, etc.

For more detailed help in scoring the fundamentals see last page of book.

below in table II show that scores for reasoning problems of grade VI pupils can be very definitely arranged in a scale on the basis of relative difficulty. Just what the scale should be can only be determined by determining the form of distribution and the location of the zero point. From what is known of these the scale of weighting shown in the last column of table II is believed to be the best, and this is the one employed in the computations of this study. However, in order to enable the reader to satisfy himself as to which is the best method, the scores of the twenty-six systems were calculated on each of three other bases—(1) counting each problem reasoned correctly a score of 1; (2) counting each problem reasoned correctly a score based on the ratio of its difficulty as shown in the next to the last column of table II; and (3) counting the scores made on only the first six problems for which presumably all pupils of all systems had ample time. See appendix, page 98.

[1]The detailed study of mistakes among the five hundred individual pupils shows this even more clearly. See table at top of p. 29.

TABLE II

PRELIMINARY TESTS

Reasoning—Unlimited Time

100 different pupils tested each time

Number of Problems.	% reasoned correctly as printed	% reasoned correctly as reversed	Average % reasoned correctly	Weight according to Ave. % correct	Weight used as probably best
1	95	92.6	93.8	1	1
2	86	82.2	84.1	1.1	1
3	94	89	91.5	1	1
4	80	83	81.5	1.15	1
5	88	86	87	1.1	1
6	69	57.4	63.2	1.5	1.4
7	70	80	75	1.25	1.2
8	29	44	36.5	2.6	1.6
9	19	15.5	17.2	5.45	2
10	24	27.4	25.7	3.6	2
11	17	7.5	12.3	7.6	2
12	7	16.4	11.7	8	2

In both reasoning and fundamentals the scores used as a measure of the achievement of a system were computed by combining the scores of one hundred pupils. Where more than one hundred pupils were tested the papers used were drawn at random, the number drawn from each class being determined by the ratio of its number to the total number tested in the system. Where less than one hundred pupils were tested, the combined scores made were raised to the basis of one hundred pupils.

PRECAUTIONS OBSERVED TO MAKE THE SCORING ACCURATE

The simplicity of the tests made the scoring comparatively easy; and with the observance of the following precautions it is believed that a high degree of accuracy was attained. (1) In so far as practicable, all the papers were scored by a single judge —only two persons being employed on any phase of the work for the entire twenty-six systems; (2) each problem was scored through one hundred or more papers, then the next followed through, etc.; (3) the score for each part of each problem, the errors, etc., were entered on a blank provided with a separate column for each item; (4) where there was doubt as to how the

score should be made, the scorer made a written memorandum of how the case was finally decided and this memorandum served as the guide for all future similar cases.

What the Scores Measure

As used in this study the words achievements, products, abilities, except where otherwise qualified, must necessarily refer to the results of the particular tests employed in this investigation. That some systems may achieve other and possibly quite as worth while results from their arithmetic work is not denied; but what is denied is that any system can safely fail to attain good results in the work covered by these particular tests. Whatever else the arithmetic work may produce, it seems safe to say that by the end of the sixth school year, it should result in at least good ability in the four fundamental operations and the simple, everyday kind of reasoning called for in these problems. It does not then seem unreasonable, in view of the precautions previously enumerated, to claim that the scores made by the respective systems afford a reliable measure of the products of their respective procedures in arithmetic.

PART II.—ARITHMETICAL ABILITIES—THEIR VARIABILITY AND RELATIONSHIP

The Problem

The first large question prompted by a preliminary survey of the achievements of the pupils in the twenty-six systems is,—*What is the nature of the product of the first six years of arithmetic work?* Part II is concerned primarily with the answer to this question as found in the results of the tests.

Some Phases of the Problem

Some of the phases of the question that a closer study of the data seems to promise help on are:

1. Is the net result of the arithmetic work of the first six years *a* product, or is it *several products?*

2. To what extent are these products uniform (a) among systems, (b) among individual pupils in the systems, (c) among boys, (d) among girls?

3. In how far does the possession of one ability imply the possession of others?

Among those who still believe in the efficacy of *formal discipline,* the possibility of there being a plurality of abilities required in the mastery of a subject such as arithmetic is scarcely entertained. Only recently has Educational Psychology shown the probability of the comparative independence and consequent lack of uniformity of capacities and products. Today, well founded doubt is entertained as to whether the possession of a given degree of ability in one phase of a subject necessarily implies the possession of anything like an equal degree of ability in another. Professor Thorndike concludes from a study of the arithmetical abilities of high school pupils[1] and related researches that "ability in arithmetic is but an abstract name for a number of partially independent abilities."[2] This part of the present study addresses itself to the furnishing of more evidence along these lines.

The Data

The source of the data used to help answer the above questions is some six thousand test papers gathered from twenty-six representative school systems. Copies of the tests may be found in Part 1, pages 10 and 11; as may also a statement of conditions under which the tests were personally given by the author, page 13; and the method of scoring, pages 15 to 18.

The achievements are considered from two standpoints—(1) the scores and mistakes of the systems as systems, (2) the scores of individual pupils as individuals.

[1] Thorndike & Fox, *Columbia University Contributions to Education*, Vol. XI, pp. 138–143

[2] Thorndike's *Educational Psychology*, p. 39.

Achievements of the Systems as Systems

Measured by Scores Made

Table III[1]

Scores of the twenty-six systems in Reasoning with deviations from the median. Scores from all problems.

M[2]=551

Systems in order of achievement	Scores made[3]	Deviations from the median	Deviations in per cent of the median
XXIII	356	−195	−35
XXIV	429	−122	−22
XVII	444	−107	−19
IV	464	−87	−16
XXV	464	−87	−16
XXII	468	−83	−15
XVI	469	−82	−15
XX	491	−60	−11
XVIII	509	−42	−8
XV	532	−19	−3
III	533	−18	−3
VIII	538	−13	−2
VI	550	−1	−.2
I	552	1	.2
X	601	50	9
II	615	64	12
XXI	627	76	14
XIII	636	85	15
XIV	661	110	19
IX	691	140	20
VII	734	183	33
XII	736	185	34
XI	759	208	38
XXVI	791	240	44
XIX	848	297	54
V	914	363	66

Table IV

Scores of the twenty-six systems in Fundamentals with deviations from the median. Scores from all problems.

M=3111

Systems in order of achievement	Scores made	Deviations from the median	Deviations in per cent of the median
XXIII	1841	−1270	−41
XXV	2167	−944	−30
XX	2168	−943	−30
XXII	2311	−800	−26
VIII	2747	−364	−12
X	2749	−362	−12
XV	2779	−332	−11
III	2845	−266	−8
I	2935	−176	−6
XXI	2951	−160	−5
II	2958	−153	−5
XVII	3042	−69	−2
XIII	3049	−62	−2
VI	3173	62	2
XI	3261	150	5
IX	3404	293	9
XII	3410	299	10
XXIV	3513	402	13
XIV	3561	450	14
IV	3563	452	14
V	3569	458	15
XXVI	3682	571	18
XVI	3707	596	19
XVIII	3758	647	21
VII	3782	671	22
XIX	4099	988	31

1 In proceeding to the part of the study that is necessarily largely composed of tables, it may be well to state the position of the author regarding the partial interpretations offered in connection with the tables. It is that the *entire tables* give by far the best basis for conclusions; that for a thorough comprehension of the study they should be read quite as fully as any other part; and that they should be regarded as the most important source of information rather than the brief suggestive readings which are liable to give erroneous impressions, both because of the limitations of a single interpretation and the lack of space for anything like full exposition.

2 M=*Median* which is the representation of central tendency used throughout this study. It has the advantages over the average of being more readily found, of being unambiguous, and of giving less weight to extreme or erroneous cases.

3 For reliability of measures of reasoning ability, see appendix p. 100.

Table III gives the scores made in reasoning by each of the twenty-six systems, counting all the problems that were solved and weighting them according to the last column of table II. The Roman numerals used in the left hand column to designate the systems are those that fell to each system by lot.[1] As seen by the column headed *scores made,* the systems are arranged according to number of scores, i. e., system XXIII made three hundred fifty-six points, the lowest score, and is placed first in the table; system XXIV made four hundred twenty-nine points, and is placed second, etc. System V, having made the highest score, is placed last in the table.

The middle column gives the deviations from the median, which is that measure above and below which one-half the cases lie. In this table the median is five hundred fifty-one. These deviations serve to show the differences in scores made; and they are also employed in computing the measures of variability and relationship. The third column is the deviations in per cent of the median. It affords another expression of the difference in size of scores made by the systems.

Table IV reads exactly as III, the scores[2] being those made on all the problems of the test in fundamentals. These two tables give some general help on the nature of the product of the first six years of arithmetic work. One very evident fact is the lack of uniformity among systems; another is the lack of correspondence of relative position among the systems in the two tables. With the exception of systems XXIII and XIV, no system occupies the same relative position in the two tables, e. g., system XXIV stands second from the lowest in reasoning and eighteenth from the lowest in fundamentals. This fact is more accurately summarized in the coefficients of correlation, table XV, page 37.

[1] See statement of Key, Part I, p. 9.
[2] As stated in Part I, p. 17, a *score* is arbitrarily set at *one*. The fact that the zero point is unknown in both reasoning and fundamentals makes these scores less amenable to ordinary handling than they might at first thought seem. Hence, entire distributions are either printed or placed on file at Teachers College.

TABLE V

Scores of the twenty-six systems in each of the Fundamental operations

Addition		Subtraction		Multiplication		Division	
System	Score	System	Score	System	Score	System	Score
XX.......	771	XXIII..	159	XXIII..	641	XXIII..	241
XXIII....	800	XX.....	216	XXV...	744	XXV...	350
XXII.....	842	XXV...	217	XX.....	763	XXII...	408
XXV.....	856	XXII...	235	XXII...	826	XX.....	418
XXI......	971	VIII....	293	XXI....	883	X......	450
VIII......	997	X......	298	VIII....	970	VIII....	487
X........	1004	III.....	299	XV.....	982	XV.....	490
XV.......	1005	XV.....	302	III.....	988	III.....	495
III.......	1063	XVII...	310	X......	997	II......	516
I.........	1063	II......	315	II......	1037	XVII...	516
XIII......	1077	I.......	317	I.......	1039	I.......	516
II........	1090	VI......	337	XVII...	1045	XIII....	547
VI........	1164	XXI....	343	XIII....	1061	VI......	558
XI........	1171	XI......	360	VI......	1113	XI.....	577
XVII.....	1171	XIII....	364	XI......	1153	XIV....	605
IX.......	1182	XII.....	377	XIV....	1189	IX.....	629
XII.......	1201	XXIV..	377	XII.....	1201	XII.....	631
XIV......	1220	IX.....	387	IX.....	1203	XXIV..	644
XXIV....	1254	IV......	392	XXIV..	1238	IV......	646
V.........	1267	XVI....	406	V.......	1239	V.......	656
XVI......	1279	V.......	407	IV......	1240	XXVI..	674
XVIII....	1280	XXVI..	409	XXVI..	1302	VII.....	695
IV........	1283	XVIII..	420	VII.....	1312	XVI....	700
XXVI....	1292	VII.....	426	XVI....	1322	XVIII..	710
VII.......	1346	XIX....	476	XVIII..	1347	XXI....	754
XIX......	1376	XIV....	547	XIX....	1433	XIX....	814

Table V gives the scores made in each of the four fundamental operations by each of the twenty-six systems. The lack of correspondence in relative position is again in evidence here, e. g., system XX stands lowest in addition, second from the lowest in subtraction, third in multiplication and fourth in division. If the net result of arithmetic work were *a* product each system would have the same relative position in each phase of the subject. However, a comparative reading of tables III, IV and V will show much more uniformity as to relative position in the fundamentals than in reasoning and fundamentals. This fact is more exactly expressed in the coefficients of correlation tables XV and XVI, pages 37 and 39.

TABLE VI

Scores of the twenty-six systems in Reasoning with deviations from the median. First six problems counted.

M=483

Systems in order of achievement	Scores made	Deviations from median	Deviations in % of M.
XXIII.......	342	−141	−29
XVII........	389	−94	−19
XVI.........	389	−94	−19
XXIV.......	396	−87	−18
IV...........	420	−63	−13
XXII........	423	−60	−12
XX..........	426	−57	−12
XXV........	438	−45	−9
III..........	445	−38	−8
XVIII.......	452	−31	−6
VI...........	455	−28	−6
I............	466	−17	−4
VIII........	468	−15	−3

TABLE VII

Scores of the twenty-six systems in Fundamentals with deviations from the Median. First six problems counted.

M=2578

Systems in order of achievement	Scores made	Deviations from median	Deviations in % of M.
XXIII.....	1776	−802	−31
XXV......	2078	−500	−19
XX........	2084	−494	−19
XXII......	2116	−462	−18
X.........	2383	−195	−8
XVII......	2416	−162	−6
I..........	2456	−122	−5
XV........	2494	−84	−3
III........	2495	−83	−3
VIII.......	2501	−77	−3
XXI.......	2548	−30	−1
II.........	2554	−24	−1
VI.........	2565	−13	−.5

TABLE VI—*Continued.*

Scores of the twenty-six systems in Reasoning with deviations from the median First six problems counted.

M=483

Systems in order of achievement	Scores made	Deviations from median	Deviations in % of M.
XIII..........	497	14	3
X..........	502	19	4
IX..........	503	20	4
XV..........	508	25	5
XIV..........	514	31	6
II..........	516	33	7
XXI..........	532	49	10
XII..........	536	53	11
V..........	549	66	14
XIX..........	564	81	17
XXVI..........	569	86	18
XI..........	576	93	19
VII..........	661	178	37

TABLE VII—*Continued.*

Scores of the twenty-six systems in Fundamentals with deviations from the Median. First six problems counted.

M=2578

Systems in order of achievement	Scores made	Deviations from median	Deviations in % of M.
XIII	2590	12	.4
IX........	2650	72	3
IV.	2694	116	4
XVIII.....	2703	125	5
XI........	2706	128	5
XXVI.....	2710	132	5
XII.......	2713	135	5
XIV.......	2717	139	5
XVI.......	2728	150	6
V.	2767	189	7
VII........	2782	204	8
XIX.......	2791	213	8
XXIV.....	2815	237	9

Tables VI and VII read precisely as III and IV and they give the same scores, except that in tables VI and VII only the scores from the first six problems are counted. The most noticeable difference between these two sets of tables is the greater uniformity of achievements in tables VI and VII; the scores of table VI vary only from three hundred forty-two to six hundred sixty-one, as compared with three hundred fifty-six to nine hundred fourteen of table III; and those of table VII vary only from one thousand seven hundred seventy-six to two thousand eight hundred fifteen, as against one thousand eight hundred forty-one to four thousand ninety-nine of table IV.[1]

[1] For a more precise statement of variability, see Table XII, p. 33.

Certain systems evidently spent much more time per problem than others. A striking example of this is system XV, which has a serial standing of *10th* from the poorest in table III and rises to *17th* from the poorest in table VI; a striking example of the change in the other direction is system XVI which has a serial standing of *7th* in table III and only *3d* in table VI. It is difficult to account for these differences in relative standing except on the basis of either economy of time[1] or accuracy of work.[2] Each of these factors will be considered.

Achievements of Systems as Measured by Mistakes[3] Made

If education is to profit by the experience of business men, reasonable accuracy in such simple, "every day" phases of arithmetic as are measured by the tests of this study should be the first consideration, i. e., by the time pupils reach high 6th it should be habitual with them to thoroughly test the work of each step before leaving it to try another. Hence, it is doubtful whether a high standing in number of scores without a reasonably good standing in accuracy is as good a rating as a fair standing in number of scores combined with a high standing in accuracy.

The next two tables give the systems in order of excellence as measured by the mistakes made.

1 Economy of time considered, pp. 62-65.
2 Accuracy of work shown, p. 27.
3 The limitations of the present study admit of only this very inadequate treatment of errors, the psychology of which is worthy of extended research study. Such studies as Brown's *The Psychology of the Simple Arithmetical Processes* [*American Journal of Psychology*, Vol. XVII, pp. 1–37]. are valuable in themselves, but conclusions based on introspection will be much safer guides to educational practice when confirmed by objective methods.

TABLE VIII

Mistakes in Reasoning

Systems in order of per cent of mistakes	Per cent incorrect	No. of problems incorrect	No. of problems attempted
XVI	45.1	359	796
XVII	44.9	335	746
XXIII	41.1	238	579
III	36.7	282	769
I	34.7	269	776
XV	33.7	249	739
VI	31.8	233	733
XXII	30.9	196	634
X	29.7	232	781
XXIV	28.9	167	577
XIII	28.8	230	799
XXVI	28.6	276	964
VIII	27.9	192	689
XVIII	27	175	648
XIX	26.4	255	965
XXV	25.3	150	592
IV	25.1	147	585
XX	23.4	139	595
VII	23.1	189	819
XIV	22.9	175	765
XII	22.3	185	831
IX	20.3	161	794
II	19.7	137	696
V	18.6	171	919
XXI	15.7	106	674
XI	14.4	112	776

TABLE IX

Mistakes in Addition[1]

Systems in order of per cent of mistakes	Per cent incorrect	No. of steps incorrect	No. of steps attempted
XXII	14.5	196	634
XX	13.5	139	595
I	13.4	115	861
XVII	10.5	96	918
XVIII	10.4	102	982
VI	10.1	92	908
X	9.9	81	819
IX	9.6	86	898
V	9.2	89	967
XIII	8.9	75	847
XV	8.8	72	818
VII	8.76	87	993
IV	8.5	81	951
XXV	8.3	61	739
XXIII	8.	56	703
VIII	7.5	60	803
XVI	7.04	67	952
II	7	60	857
III	6.5	55	843
XII	6.3	56	896
XIV	6.2	57	921
XXIV	5.9	54	918
XXVI	5.8	54	930
XIX	5.78	57	987
XXI	5.1	44	860
XI	4.7	42	888

1 The mistakes of the addition problems *only* were used as a measure of accuracy in fundamentals. Compare errors in each of the four fundamentals in four systems, p. 29.

In these tables the systems are arranged in the order of percentages of mistakes made, i. e., system XVI made the largest percentage of errors in reasoning and system XXII made the largest percentage in fundamentals. Hence system XVI appears at the beginning of table X and system XXII at the beginning of table XI. Reading the first line of table VIII we have,—system XVI made 45.1% of errors in reasoning, having attempted 796 problems and having reasoned 359 of these incorrectly. A glance down the first column of the table shows that the twenty-six systems varied in reasoning mistakes fairly regularly from 45.1% to 14.4%.

The second and third columns of this table are given not only to show the method of computing the measure of accuracy but also to show that the *number* of errors is not a sufficient basis upon which to judge accuracy, e. g., system XIX which, according to the percentage of errors made, ranks 15*th* from the poorest in accuracy of reasoning, made 255 errors which, on the basis of the *number* of errors made, would place it only 6th from the poorest.

This table is worthy of further study in comparison with table III, by which it is seen that the standings of the systems in rapidity and accuracy do not correspond. Two marked examples of excellence in accuracy are system XI which stands 1st in accuracy and 4th from the best in rapidity, and system XXI which stands 2d in accuracy and 10th from the best in rapidity.

Table IX, which gives the systems in order of excellence as to accuracy in addition, is made up on the same basis as table VIII and reads precisely the same. As a measure of accuracy in fundamentals, addition has the following advantages,—(1) as shown in tables XVI and XVIII, pages 39 and 41, addition is the least like reasoning of any of the fundamentals; and (2) as is shown below, the percentages of errors in addition exceed those for any other fundamental operation.

Percentages[1] of errors in each of the four fundamentals of four systems selected at random:

System	Add.	Sub.	Mul.	Div.
XVII	4.6	4.9	4.7	3.7
XI	3.5	2.2	1.5	.7
VIII	6.1	7.3	3.9	.6
XIV	4.5	2.1	1.4	.5
Average	4.7	4.1	2.9	1.4

The percentages of errors found in the fundamentals of these four systems show clearly what was suggested by table I, viz., that addition is not only just as hard as any other fundamental for grade VI pupils but even harder than the others to do correctly. The difficulty of accuracy seems to decrease from addition through substraction and multiplication and become least in division. Hence, addition seems the best of the four fundamental operations as a measure of accuracy of systems in the more formal phases of arithmetic.

It is worth while to compare tables VIII and IX for the lack of correspondence in excellence as to accuracy in reasoning and accuracy in addition. One sees at a glance that only two systems, XI and XXI, did equally well in both and that many systems differ very widely.[2]

Achievements of Pupils as Individuals

As the most exact knowledge of the nature of the product of the first six years of arithmetic work is to be gained by a study of the achievements of individual pupils rather than of groups, the individual scores made by five hundred pupils are given in the following tables.

[1] The *base* used in figuring percentages of mistakes for each of the fundamentals was the number of steps attempted.

[2] This lack of correspondence might be expressed more precisely in coefficients of correlation as is done for the lack of correspondence shown in Tables III and IV (see Table XV). Taking the twenty-six systems as a group, the coefficient would be comparatively little above zero.

TABLE X

Scores made in reasoning by 500 pupils chosen at random from four representative public school systems:

Scores[1]	XVII		XI		VIII		XIV		Scores[1]	XVII		XI		VIII		XIV	
	50 boys	50 girls	50 boys	50 girls	50 boys	50 girls	100 boys	100 girls		50 boys	50 girls	50 boys	50 girls	50 boys	50 girls	100 boys	10 girls
0	3	3	....	....	....	....	1	2	78	2	....	2	1	1	....	5	1
10	1	2	....	....	....	4	....	2	79	....	....	....	....	....	....	....	....
13	....	....	....	....	....	....	1	....	80	1	1	2	2	2	1	2	....
18	....	....	....	....	....	....	1	....	81	....	....	....	....	....	1	....	....
20	3	4	....	1	2	....	1	2	82	....	....	1	....	1	....	2	4
22	....	1	....	....	....	....	....	2	83	....	....	....	....	....	....	....	....
23	1	....	....	....	....	....	....	1	84	....	....	....	1	....	1	....	....
24	....	1	....	....	....	....	....	....	85	....	....	....	....	....	....	....	....
25	....	....	....	....	....	....	....	1	86	....	1	....	....	....	....	1	1
26	....	....	....	....	....	1	....	....	87	....	....	....	....	....	....	....	....
27	....	....	....	....	....	....	....	....	88	....	....	....	....	....	....	....	....
28	....	....	....	....	....	....	1	....	89	....	....	....	....	....	....	....	....
29	....	....	....	....	....	....	....	....	90	....	....	....	....	....	....	....	....
30	11	5	....	....	....	4	1	6	91	....	....	....	....	....	....	....	....
31	....	....	....	....	....	....	....	....	92	....	....	3	5	4	....	12	4
32	....	....	....	....	....	....	1	2	93	....	....	....	....	....	....	....	....
33	....	....	....	....	1	1	....	....	94	....	....	....	....	....	....	....	....
34	....	1	....	1	1	....	....	....	95	....	....	....	....	....	....	....	....
35	....	....	....	....	....	....	....	....	96	....	....	....	1	....	....	2	....
36	....	1	....	....	....	....	....	3	97	....	....	....	....	....	....	....	....
37	....	....	....	....	....	1	....	....	98	....	....	2	....	....	....	3	....
38	....	....	....	....	....	....	....	....	99	....	....	....	....	....	....	....	....
39	....	....	....	....	....	....	....	....	100	....	1	....	....	....	....	....	....
40	5	3	....	....	6	10	1	10	101	....	....	....	....	....	....	....	....
41	....	....	....	....	....	....	....	....	102	....	....	3	2	1	1	1	....
42	....	....	....	....	....	....	1	2	103	....	....	....	....	....	....	....	....
43	....	....	....	....	....	1	....	....	104	....	....	....	....	....	....	....	....
44	1	1	....	....	1	1	....	....	105	....	....	....	....	....	....	....	....
45	....	....	....	....	....	....	1	....	106	....	....	....	....	....	....	....	....
46	2	2	....	....	....	3	....	....	107	....	....	....	....	....	....	....	....
47	....	....	....	....	....	1	....	1	108	....	....	....	....	....	....	....	....
48	....	....	....	1	1	....	....	1	109	....	....	....	....	....	....	....	....
49	....	....	....	....	1	1	....	....	110	....	....	....	....	....	....	....	....
50	3	3	5	5	4	2	8	9	111	....	....	....	1	....	....	1	....
51	....	....	....	....	....	1	....	....	112	....	1	3	1	....	....	5	1
52	1	3	1	1	....	....	5	7	113	....	....	....	....	....	....	....	....
53	....	....	....	....	....	1	....	....	114	....	....	....	....	....	....	....	....
54	3	5	3	4	3	4	2	4	115	....	....	....	....	....	....	....	....
55	....	....	....	....	....	....	....	....	116	....	....	1	....	....	....	....	....
56	1	....	....	....	1	1	3	5	122	1	....	....	....	....	....	....	....
57	....	....	....	....	....	....	....	....	127	....	....	....	....	....	....	1	....
58	....	....	....	1	....	....	3	1	...	....	....	....	....	....	....	....	....
59	....	....	....	1	1	....	....	....	132	....	....	1	....	....	....	....	1
60	....	2	....	1	1	1	2	2	...	....	....	....	....	....	....	....	....
61	....	1	1	....	1	....	....	1	143	....	....	1	....	....	....	....	....
62	1	....	....	....	1	1	9	5	...	....	....	....	....	....	....	....	....
63	....	....	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
64	4	2	11	3	7	3	7	6	152	....	....	1	....	....	....	....	....
65	....	....	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
66	....	....	1	4	3	2	4	5	...	....	....	....	....	....	....	....	....
67	....	....	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
68	....	....	....	....	....	....	....	2	...	....	....	....	....	....	....	....	....
69	1	....	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
70	....	2	....	1	3	....	3	....	...	....	....	....	....	....	....	....	....
71	....	....	....	....	2	....	....	....	...	....	....	....	....	....	....	....	....
72	1	....	....	2	....	....	1	....	...	....	....	....	....	....	....	....	....
73	....	....	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
74	1	1	....	....	....	....	....	....	...	....	....	....	....	....	....	....	....
75	....	....	....	10	....	....	....	....	...	....	....	....	....	....	....	....	....
76	3	3	8	....	1	2	8	5	...	....	....	....	....	....	....	....	....
77	....	....	....	....	....	....	....	1	...	....	....	....	....	....	....	....	....

[1] Scores given in tenths, and decimal points omitted. For reliability of scores, see appendix, p. 100.

TABLE XI

Scores made in Fundamentals by 500 pupils chosen at random from four representative public school systems.

Scores	XVII		XI		VIII		XIV	
	50 boys	50 girls	50 boys	50 girls	50 boys	50 girls	100 boys	100 girls
3	1	......	......	......	......	......	......	......
9	......	......	......	......	......	2	......	......
11	4	1	......	......	......	1	......	......
12	......	......	......	......	1	1	......	1
13	2	1	......	......	......	1	......	......
14	1	......	......	1	1	1	......	2
15	1	2	2	......	1	1	......	......
16	1	.	......	......	1		......	2
17	1	......	......	......	2	......	......	3
18	1	......	......	......	......	2	1	1
19	2	1	......	4	4	2	4	3
20	2	5	2	......	2	3	1	2
21	1	3	1	......	2	2	2	6
22	1	2	......	1	1	......	1	3
23	1	3	......	2	3	2	2	2
24	1	1	2	....	1	3	1	2
25	......	1	1	1	2	......	1	1
26	4	2	1	......	......	2	2	3
27	2	1	3	3	1	4	3	3
28	2	1	1	4	3	2	1	3
29	1	2	2	3	5	3	7	7
30	1	5	7	8	7	4	13	7
31	......	4	......	4	3	1	6	2
32	2	......	......	1	3	3	4	2
33	2	2	6	1	1	2	4	7
34	3	1	4	1	1	1	1	5
35	1	1	......	3	......	2	2	1
36	1	1	......	......	1	1	1	......
37	......	......	......	......	......	......	3	1
38	......	1	2	......	......	1	1	2
39	1	2	1	2	1	......	3	4
40	1	1	2	4	....	1	6	......
41	2	1	2	2	2	1	3	......
42	......	1	2	......	1	......	1	1
43	......	......	......	......	......	......	3	1
44	1	1	......	......	......	......	2	2
45	......	1	1	......	......	......	3	1
46	1	......	1	......	......	1	2	3
47	1	......	2	1	......	......	3	4
48	1	1	2	1	......	......	1	3
49	......	......	......	1	......	......	3	3
50	......	......	......	2	......	......	5	3
51	1	1	......	......	......	......	......	1
52	......	......	......	......	......	......	......	......
53	1	......	......	......	......	......	2	1
54	......	......	2	......	......	......	......	1
55	1	......	......	......	......	......	1	......
56	......	......	1	......	......	......	......	......
57	......	......	......	......	......	......	......	......

TABLE XI—*Continued*

Scores	XVII		XI		VIII		XIV	
	50 boys	50 girls	50 boys	50 girls	50 boys	50 girls	100 boys	100 girls
58	......	......	......	......	......	......	......	1
59	......	......	......	......	......	......	......	......
60	......	......	......	......	......	......	......	......
61	......	......	......	......	......	......	......	......
62	......	......	......	......	......	......	......	......
63	......	......	......	......	......	......	1	......

Table X shows the individual scores made by each of five hundred pupils in reasoning, the pupils being taken at random from four systems which were also chosen quite at random from among the twenty-six, except that they are all public schools.

The scores were computed exactly as for the system measures of tables III and IV, i. e., according to the weighting of the problems given in the last column of table II, and the scores are given separately for boys and girls. The left hand column of each half of the table gives the possible scores from 0 to 15.2. Reading for system XVII, three boys and three girls made no score, one boy and two girls made a score of only 1, three boys and four girls made 2, etc.

Table XI is made up for fundamentals on the same basis as table X and is read in precisely the same way.

Doubtless the most noteworthy feature of these tables is the wide variability of achievements. That pupils of high 6th should vary in standing from zero (the exact value of which is unknown) to 15.2, that nine out of five hundred pupils should fail to solve any of the simple problems of the reasoning test, and that with the median score about six, thirty-three should make a score of only two or less, raises the question of the proper care of the mentally deficient, but more directly the question of flexibility in grading and sectioning. Is it right from any standpoint to expect to treat children as if they could and ought to do equally well in all subjects, or even in all phases of the same subject?

Of the four systems here represented, the pupils of XVII show the greatest variability in both reasoning and fundamen-

tals, and hence as measured by abilities in arithmetic are probably graded least well.

Variability of Abilities

As there is no single figure by which variability can be so expressed that comparisons are entirely reliable, the best source for determining variabilities is the distributions themselves as given in the preceding tables; but in order to afford a more available form for comparison the following measures were computed.

TABLE XII

Variability among systems and among individual pupils in terms of average deviation from the median and in coefficients of variability. Computed from Tables III, IV, VI, VII, X, and XI.

	Reasoning		All fundamentals		Addition		Subtraction		Multiplication		Division	
	AD[1]	Co^t	AD	Co^t	AD	Co^t	AD	Co^t	AD	Co^t	AD	Co^t
26 Systems—all problems	112.2	.20	420.7	.13	139.2	.12	68.8	.20	166	.15	105.5	.19
26 Systems—1st 6 problems	58.3	.12	188.4	.07	.....	....	.....	....	.....	.....	.....	....
100 pupils, XVII	1.98	.43	8.53	.30	3.41	.32	1.59	1.06	3.15	.34	2.17	.58
100 pupils, XI	1.66	.22	6.95	.22	3.19	.28	1.05	.35	2.42	.23	1.41	.29
100 pupils, VIII	1.56	.29	6.21	.22	2.34	.27	1.33	.60	1.60	.18	1.49	.35
200 pupils, XIV	1.93	.30	7.94	.25	3.30	.29	1.35	.41	2.87	.27	2.03	.37
Average for individual pupils of four systems	1.76	.32	7.41	.25	3.06	.29	1.33	.61	2.51	.26	1.78	.40

This table gives help in comparing variability in several ways:

(1) The first line, reading across the page, shows the variability among systems in the different phases of the subject. The coefficients range from .12 in addition to .20 in reasoning and subtraction. This more uniform achievement in addition may be due to a more generally accepted idea of what work in addition ought to be and a more uniform determination to get

[1] A D=average deviation, and Co^t=coefficient of variability. The method by which the A. D's or average deviations are here computed is to divide the sum of all the deviations from the median (without regard to signs) by the number of cases; the coefficients of variability are here determined by dividing the A. D's by the median.

it up to standard, or possibly to a more uniform ability on the part of groups of pupils taken group by group to do addition, or to a combination of these factors. The coefficient of .13 for fundamentals as compared with .20 for reasoning is also noteworthy, as indicating that systems differ less widely in achievements in the more mechanical processes than in reasoning.

(2) The second line is important in that it shows the same general tendency toward more uniformity in fundamentals and in that the coefficients for both reasoning and fundamentals are so much smaller. This last is what might be expected and is proof that the coefficients measure variability, for it stands to reason that all systems would be much closer together when the scores were taken from only the first six problems on which practically all pupils of all systems had time to work.

(3) The last five lines deal with variability among individual pupils. The coefficients for the four systems indicate some marked differences in degree of uniformity of pupils. As is evident from the distribution tables, system XVII has the greatest variability, exceeding all others in all phases of the subject. Fifteen of the one hundred pupils made no score in subtraction and twenty-nine made a score of only *one* each. It may be noted in this connection that in this system there is no supervision of arithmetic work, except certain official testing by the superintendent.

Another noteworthy fact is that the coefficients are higher in each of the four systems for reasoning than for fundamentals, due probably to fundamentals being more readily taught than reasoning, i. e., to reasoning being more dependent on original ability.

The last line gives measures of the variability among five hundred pupils for each of the phases of the subject. If the subtraction coefficient, which is so very high because of system XVII, be omitted it is noticeable that pupils vary most in division and next in reasoning, and about equally in addition and multiplication.

This agrees in general with the findings of Thorndike and Fox,[1] whose coefficients show more variability for both boys and girls in reasoning than in addition and multiplication.

1 *Columbia University Contributions to Philosophy, Psychology & Education*, Volume XI, p. 145.

TABLE XIII

Variability among boys and among girls in terms of average deviation from the median and in coefficients of variability computed from tables VIII and IX, and from others on file in the library of Teachers College.

Systems	Reasoning		All fundamentals		Addition		Subtraction		Multiplication		Division	
	AD	Co^t	AD	Co^t	AD	Co^t	AD	Co^t	AD	Co^t	AD	Co^t
XVII 50 boys	1.91	.42	9.96	.37	3.46	.33	1.64	.94	3.48	.37	2.30	.56
XVII 50 girls	1.97	.39	7.34	.25	3.34	.29	1.48	.99	2.81	.30	2.00	.57
XI 50 boys	1.82	.24	7.54	.23	3.17	.26	1.14	.38	2.64	.25	1.54	.31
XI 50 girls	1.48	.21	6.18	.21	2.80	.31	.92	.29	2.13	.21	1.31	.26
VIII 50 boys	1.43	.23	5.70	.20	3.34	.25	1.36	.39	1.98	.22	1.59	.33
VIII 50 girls	1.39	.30	6.64	.25	2.56	.30	1.20	.48	2.14	.23	1.52	.37
XIV 100 boys	1.81	.27	9.91	.24	3.05	.25	1.13	.38	3.07	.28	2.09	.38
XIV 100 girls	1.74	.33	8.87	.30	3.29	.35	1.35	.43	2.00	.28	2.14	.40

For this table the same data and the same formulae were used as for the coefficients among individuals in table XII; but here the variability for boys and for girls is given separately. As would be expected, all the coefficients for system XVII are the highest. The chief value of this table is that it affords a basis for comparing boys and girls in variability. The next table is composed of the ratios of the coefficients of the boys to the coefficients of the girls.

TABLE XIV

Ratio of variability of boys to variability of girls. Computed from coefficients of the last table above.

Systems	Reasoning	All Fundamentals	Addition	Subtraction	Multiplication	Division
XVII	1.08	1.48	1.14	.95	1.23	.98
XI	1.14	1.10	.84	1.31	1.19	1.19
VIII	.77	.80	.83	.81	.96	.89
XIV	.82	.80	.71	.88	1.00	.95
Average for four systems	.95	1.05	.88	.99	1.10	1.00
P. E.	.07	.10	.05	.07	.05	.04

This table shows that for the first two systems, viz., XVII and XI, the boys are somewhat more variable, and in systems VIII and XIV about the same amount less variable. This is interesting and points to a need for further investigation, for the common opinion is that men are more variable than women; and supposedly boys more so than girls. But as seen by the averages for these four systems, so far as these 250 boys and 250 girls show the true tendency, there are no more exceptionally bright or exceptionally dull pupils among the boys than among the girls at this age. The probable error of each of the coefficients is given in the last line which reads, for reasoning the chances are equal that if an infinite number of cases were measured, the boys would not be more than 102% nor less than 88% as variable as the girls, etc.

Relationship of Abilities

The most exact answer to the question, *how far does the possession of one ability imply the possession of others,* is found by computing coefficients of correlation. In reading tables III and IV, it was noted that the systems did not do equally well in each of the phases of arithmetic. That is to say, these tables show at a glance that a given ability in reasoning is not necessarily accompanied by the same degree of ability in fundamentals. As was pointed out, only two systems, XXIII and XIV, have the same relative position in both tables, while several systems have decidedly different standings. The reading of tables X and XI shows even wider variability among individual pupils. The extent of such differences is best measured by the coefficient of correlation. As employed in this part of the study, it measures degrees of kinship among abilities; and in Parts III and IV, it indicates the degree to which certain factors, supposed to produce abilities, are functioning in present practice. For the benefit of the reader unacquainted with its nature, the following explanation is adapted from Thorndike's *Educational Psychology,* pages 25 to 27.[1] The coefficient of correlation is " a single figure so calculated from the individual records as to give the degree of relationship between the two traits which will best account for all the separate cases in the group. In other words it expresses the degree of relationship from which the actual cases might have arisen with least improbability. It has possible values from

[1] A full explanation of the meaning and methods of calculating coefficients by the Pearson method may be found in Dr. Thorndike's *Mental and Social Measurements*, chapter IX.

\+ 100 per cent through 0 to — 100 per cent." A coefficient of correlation between two abilities of + 100 per cent would mean that the best system or pupil in the group in one ability would be the best in the other, that the worst system or pupil in the one would be the worst in the other, that if the individuals were ranged in order of excellence in the first ability and then in order of excellence in the second, the two rankings would be identical, that any one's station in the one would be identical with its station in the other (both being reduced to terms of the variabilities of the abilities as units to allow comparison). A coefficient of — 100 per cent would *per contra* mean that the best system or pupil in the one ability would be the worst in the other, that any degree of superiority in the one would go with an equal degree of *inferiority* in the other, and vice versa. "A coefficient of + 62 per cent would mean that (comparison being rendered fair here as always by reduction to the variabilities as units) any given station in the one trait would imply 62 hundredths of that station in the other. A coefficient of — 62 per cent would of course mean that any degree of superiority would involve 62 hundredths as much inferiority, and vice versa."

TABLE XV

RELATIONSHIP OF ABILITIES AMONG SYSTEMS

Coefficients of Correlation: Reasoning with Fundamentals—Twenty-six systems except where noted. Computed from data of Tables III, IV and V.

	Pearson	P E[1]	Median Ratio	Cosine $\pi\omega$	Average
Reasoning with all Fundamentals . .	.557	.09	.281	.368	.40
Reasoning with all Fundamentals—23 systems[2]	.731	.07	.797	.674	.73
Reasoning with all Fundamentals—15 public school systems[3]	.552	.12	.798	.809	.72
Reasoning with Addition	.398	.11	.192	.368	.32
Reasoning with Subtraction	.627	.08	.315	.562	.50
Reasoning with Multiplication	.527	.10	.385	.368	.43
Reasoning with Division	.550	.09	.345	.562	.49

[1] Reliabilities were figured for the Pearson coefficients. To illustrate the reading, the chances are about *one* to *one* that the true correlation for reasoning with all fundamentals will not vary from .56 by more than .09, i. e. it will not rise above .65 or fall below .47.

[2] This correlation omits Systems XVII, XXIV, and XVI, each of which follows the avowed policy of placing the main emphasis on the fundamentals.

[3] This correlation omits the three above named systems and all others except *public school systems*.

The Pearson, Median Ratio and Cosine $\pi\omega$ are three methods of determining coefficients of correlation. Each method has its special advantages, according to number of cases, form of distribution, etc. All three methods[1] have been used in dealing with the relationship among systems. The average is probably the most reliable and it is the measure used except when otherwise specified. For a full treatment of the relative advantages of the respective methods, see Professor Thorndike's *Empirical Studies,* page 25.

As might be expected, none of these coefficients are negative, but on the other hand all are somewhat surprisingly low. When the common practice of endeavoring to get pupils to do equally well in all phases of arithmetic is taken into account, one might expect that the pupils of these different systems would come fairly near the same level in the different phases of the subject, whereas none of the average coefficients is above .50 when all twenty-six systems are used.

Unfortunately there are, so far as the author is aware, no other coefficients of correlation showing relationships among systems. In order to get a basis for comparison, an endeavor was made to determine certain of the systems whose teaching was aimed at some particular phase of arithmetic to the partial exclusion of others. Systems XVII, XXIV and XVI were found to place the first emphasis on the fundamentals during the fourth and fifth and somewhat in the sixth grade; they were dropped out and the coefficients computed as they appear in the second line of this table. Then for further purposes of comparison these three and all others but fully represented public systems were omitted and the coefficients in the third line resulted.

These large increases in the size of the coefficients when the systems that avowedly try to teach fundamentals best are omitted can hardly be explained except on one basis, viz., that these coefficients measure teaching; and from this and the added evidence found in the fact that all the coefficients of this table are higher than the corresponding ones of table XVII, it is concluded that coefficients of correlation among systems measure, primarily, teaching rather than individual abilities of pupils. These coefficients probably show the relative success with which

[1] The author accepts these methods on authority, disclaiming a knowledge of the mathematics on which these and the other formulae of this study are based.

teachers succeed in getting all phases of arithmetic, measured by the tests of this study, equally well taught; or, put in other words, they probably measure how far teachers are at present getting their groups of pupils to show the same general average in each phase of the arithmetic, measured by the tests of this study.

How far such equality of achievement ought to prevail is an open question, one which may need a somewhat different settling for different school systems; but the author ventures the opinion that, unless it is fairly certain that the pupils are going to remain in school long enough after Grade VI for the deficiencies to be made up, care should be taken to see to it that as thorough grounding as is in harmony with the varying individual capacities, be given each pupil in such straightforward computing and such everyday reasoning as is demanded by the tests on which the above correlations are based.

TABLE XVI

Coefficients of Correlation: Fundamentals with Each Other—Twenty-Six Systems. Computed from data of Table V.

	Pearson	P. E.	Median Ratio	Cosine $\pi\omega$	Average
Addition with Subtraction.........	.869	.03	.909	.968	.92
Addition with Multiplication.......	.933	.02	.956	.968	.95
Addition with Division............	.805	.04	.919	.968	.90
Subtraction with Multiplication....	.877	.03	.999	.968	.95
Subtraction with Division.........	.841	.04	.971	.968	.93
Multiplication with Division.......	.863	.03	.943	.968	.92

The high correlations among the various fundamentals have been anticipated in tables I and V. While the difference is not large, it is worth noting that addition with division is the lowest. This same tendency is seen in the tables of correlations among individuals, where division runs highest with reasoning and higher with subtraction and multiplication than with the more mechanical processes of addition. This raises the question of whether division is not considerably more akin to rea-

soning than addition; and *a priori* it does not seem unreasonable when one considers that the only part of the division process that makes it division is *thinking* the correct quotient figure. If subsequent study proves this to be true empirically, it would seem that the notion of a sharply drawn line between the fundamentals as formal, and reasoning as thought work, must be given up.

The next two tables help most to answer the question, *how far the possession of one ability implies the possession of others.*

TABLE XVII

Coefficients of Correlation; Reasoning with fundamentals—500 individual pupils, selected at random from four representative public school systems. Cosine $\pi\omega$ method used. Computed from tables X and XI.

System	XVII 100 pupils	XI 100 pupils	VIII 100 pupils	XIV 200 pupils	Average
Reasoning with all Fundamentals.........	.156	.279	.397	.467	.32
Reasoning with Addition..............	.062	.309	.338	.425	.28
Reasoning with Subtraction............	.338	.156	.368	.411	.32
Reasoning with Multiplication..........	.309	.218	.368	.467	.34
Reasoning with. Division...............	.187	.309	.481	.467	.36

As noted above these coefficients are lower than those correlating the abilities of groups of pupils by systems. This means that however hard the systems may be trying, they are not getting all the pupils up equally well along all lines, and that it is doubtful if they should. As noted again in Part III, page 65, the decided differences among individuals in ability to do the different phases of arithmetic equally well, raise the question of the economy of having them recite in the same groups in the different phases of the subject. It even raises the question of whether teachers ought to try to get all to achieve equally well in all of even the simpler phases of the subject; and it sug-

gests the advisability of excusing some outright from the more difficult phases.

TABLE XVIII

Coefficients of Correlation: Fundamentals with each other—500 individual pupils, selected at random from four representative public school systems.

System / Coefficients by Cosine $\pi\omega$	XVII 100 pupils	XI 100 pupils	VIII 100 pupils	XIV 200 pupils	Average
Addition with Subtraction	.368	.397	.562	.674	.50
Addition with Multiplication	.637	.612	.637	.718	.65
Addition with. Division	.481	.612	.562	.574	.56
Subtraction with Multiplication	.876	.876	.904	.897	.89
Subtraction with Division	.951	.960	.951	.923	.95
Multiplication with Division	.904	.975	.951	.955	.95

This table again shows that individuals are not so nearly equal in abilities as are systems. The low correlation between addition and subtraction is surprising if one believes that subtraction is but the reverse of addition. These low correlations may be due to the fact that a majority of teachers still teach subtraction as a separate operation. So far as the author has been able to learn there have been no previous studies of the relationships of the fundamentals, other than addition and multiplication.[1] Table XVIII suggests a gradually increasing kinship that may be accounted for by the increase of reasoning involved. This hypothesis is borne out in that addition correlates best with multiplication which is evidently most like it in its mechanical nature; and it correlates least well with subtraction and not much better with division. In accord with this line of reasoning, it would be expected that the highest correlation would be be-

[1] The study of Thorndike & Fox contains correlations of addition, multiplication and different phases of reasoning. These correlations agree in general though they run somewhat higher. They were computed from tests made on high school pupils.

tween subtraction and division as they in common demand more reasoning. This is found to be the case; the coefficient of .946 for subtraction with division, is only equalled by the coefficient for multiplication and division. Not too much can be based on this single study, but the increase from .50 to .95 is worthy of note by future investigators, especially as it is a constant difference among the five hundred pupils. Another notable feature of this table is the remarkable agreement of the coefficients for the different systems.

It may be well to recall here the fact that the method of scoring as shown in table I, page 16, gave counts for each performance of each fundamental, whether the addition were in an addition or a multiplication problem, and whether the multiplication were in a multiplication or a division problem. That is, the score was given for what the operation was, wherever it occurred. This with the coefficients of the last table above make it seem safe to say tentatively of the fundamentals that the possession of ability in addition is the least guarantee of the possession of ability in others; that the possession of ability in multiplication is the best guarantee of the possession in others; and that this probably means that multiplication is like addition on its mechanical side and like division on its thinking side. Hence if one wished to measure abilities in fundamentals by a single test, one in multiplication would be best; and a test in division would probably be the best single measure of arithmetical abilities.

Concerning the question of how far the possession of ability to reason implies ability to do fundamentals, table XVII shows that it probably implies a trifle more in division than in any other fundamental, but on the whole, ability in reasoning implies ability in fundamentals no more than ability in such a subjects as English implies ability in mathematics in general, and not so much as ability in English implies ability in such subjects as geography and history.[1]

Summary

The single word that best describes the nature of the product of the first six years of arithmetic work is *complex*.

[1] Compare coefficients by Smith, Burriss and Parker—Quoted by Thorndike, *Educational Psychology*, pp. 36–37.

Taking up the phases of the problem in the order formulated:

1. The net result of the arithmetic work of the first six years is *several products* rather than *a product*. The study called arithmetic makes demands on a plurality of abilities. Hence it is inaccurate to speak of the arithmetical ability of pupils, and it is bad educational practice to treat the subject as though it were a unity instead of a plurality.[1]

2. (a) The decided lack of uniformity among systems is seen (1) in that products vary in *amounts* from 356 to 914 with an average deviation of 112 in reasoning, and from 1841 to 4099 with an average deviation of 421 in fundamentals; (2) in that products vary in *accuracy* from 45.1% to 14.4% of the problems attempted in reasoning, and from 14.5% to 4.7% of the steps attempted in addition. (b) The variability among individuals within a system is even greater than that among systems. (c) + (d) The variability among boys does not appreciably differ from that among girls.

3. The possession of a certain amount of ability by a system is a better guarantee of the possession of the same amount of another ability by that system, than the possession of a certain amount of ability by an individual, that he will have the same amount of another ability. Ability in any fundamental except addition implies nearly the same ability in other fundamentals in both systems and individuals; but ability in any fundamental implies ability in reasoning in individuals to a less degree than ability in such a subject as English implies ability in such a subject as geography. And the relationship among systems is only a little closer. Of the fundamentals, division seems to be most like reasoning, perhaps subtraction next, then multiplication, a close third, and addition least of all.

Factors Determining Abilities

Inherited Capacities.—With the possible single exception of one system, the pupils tested came from a sufficiently cosmopolitan stock so that it is believed that inherited capacities were equal.

Maturity.—The grade tested is believed to account for this factor.

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1 For a very clear and definite arrangement of the types of reasoning for the first five grades, see Prof. Suzzallo's article on *Reasoning in Primary Arithmetic, California Education*, June, 1906.

Environment.—Environment probably has little effect on arithmetical abilities. Of the five highest systems, the majority of pupils of one came from a crowded tenement district, those of two from exceptionally good homes, and those of two from fair. Practically the same distribution is found among the five systems standing lowest.

The So-Called Methods of Arithmetic.—Of the one hundred twenty teachers whose pupils were tested in this study only *eleven* said they were following any special method such as Speer, Grubé or Spiral. The potency of these so-called methods needs testing; it can probably best be done in grade IV.

Differences in quality of teaching.—This is accounted for, except in so far as the entire teaching body of one system differs from that of another, (a) by giving tests to advanced 6th grade classes which represent the teaching of a large number of teachers, [See table XIX below.], (b) by giving the tests to all the advanced 6th classes in a representative number of buildings of each system, (c) by the nature of the tests. They are meant to be sufficiently broad and sufficiently general to avoid paralleling the immediate or remote work of any particular teacher.

TABLE XIX

Number of teachers whose teaching is represented in Grade VI classes. Based on three fairly full Grade VI classes, chosen at random among the classes of three representative systems.

	No. Pupils	No. Teachers
System X	39	14
System XII	15	21
System XI	26	61
	—	—
Total	80	96
Average	27	32

1.2 teachers for each pupil

From this table and from the number of buildings tested it is estimated that the tests of this study measure the teaching of at least thirty to sixty teachers in the private or single building systems; and of fifty to one hundred fifty teachers in the public school systems.

Supervision.—See table XX below. This grouping of systems was made on the basis of answers of superintendents to the questionnaire found on page 94. The scores used are those of tables III and IV, page 21.

TABLE XX

Average Achievements of Systems Classified on Basis of Supervision.

N. B. The median achievement for all systems in Fundamentals =3111; in Reasoning, 551	Number of Systems	Average Score		Average Deviation	
		Reasoning[1]	Fundamentals[1]	Reasoning	Fundamentals
Supervision		17.5	15.5		
Supt. or Superv.	11	644	3312	105	302
		18.5	17.5		
Supt. or Superv. & Prin.	7	680	3447	108	237
		14	9		
Prin. only	2	584	2914	52	135
		7.5	13.5		
None	2	497	3108	53	65
Test		14	14.5		
By Superv. or Supt.	11	593	3221	92	287
None or only by Prin.		18	14		
or Teachers	6	651	3183	116	328

This table shows that the systems in which there was supervision by superintendent or supervisor did better than those without this supervision and that the best work was done with the supervision of both supervisor and principal. But the second part of the table indicates that the conclusion of Dr. Rice as to the extreme potency of tests does not hold in present practice. This table also shows that the systems in which demands were made by the supervisors in the form of tests did a little better than those without these demands in fundamentals but considerably poorer in reasoning. This is what might be expected to result unless a supervisor was very careful as to the kind of tests given.

While the number of systems as here divided is too small and the ratings as to supervision too inaccurate to warrant any but tentative conclusions, table XX is, so far as the author knows, the best data there is and, on its face, there is no reason to

[1] Lower numbers = average score; upper = average serial standing according to the relative position of the average score among the scores of the twenty-six systems.

believe that supervision is functioning any more generally than any other factor in producing abilities.[1]

The Expenditure.—This constitutes Part III of the study.

The Course of Study.—This is Part IV of the study.

PART III.—ARITHMETICAL ABILITIES AND TIME EXPENDITURE

The Problem

The purpose of this part of the study is the consideration of time expenditure as a factor in producing arithmetical abilities. The main problem is the determination of the relation of time expenditure to arithmetical abilities. To what extent does time expended signify abilities produced, is one way of putting the question. This inquiry is being anxiously made in connection with the feeling that the elementary school curriculum is seriously overcrowded. 'What is becoming of the *fundamentals*' is a question that is very properly being insisted upon not only by the friends of the instruction of earlier days, but quite as earnestly by many who would like to be friendly to the sincere efforts of present day innovators.

An Analysis of the Problem

The main steps in the solution of this problem are the determination of (1) present products, (2) present cost and (3) the relation of product to cost.

It is evident, that a full answer to the query 'what is becoming of arithmetic as a fundamental' would include not only the above steps but another, viz., the determination of what *ought to be* the product of arithmetic work. While this is a question that needs answering, one worthy of investigation by research, it is beside the present study. However, as previously stated,[2] it is here assumed that whatever else arithmetic work should pro-

[1] The potency of supervision needs investigation. Such a study is being made at Teachers College by Mr. C. H. Elliott.

[2] Part I, page 19.

duce, it can not afford to fail to produce reasonable proficiency in the kind of work of which these tests are composed.

Part II has dealt with abilities as measured by these tests and Part III will be devoted to time expenditure as a factor in the production of abilities.

Time Expenditure

So far as the author is aware Payne's[1] was the first comprehensive investigation of time allotments. Dr. Payne found that among ten leading American cities there was a variation in the time devoted to arithmetic from 12% of the school time in New York City to 19.5% in Jersey City, with an average of 17.3% for all the cities, which included beside the above, Boston, Chicago, Cleveland, Columbus (Ga.), Kansas City, Louisville, New Orleans and San Francisco. In an historical part of the study, he gives the following percentages of time allotted to arithmetic at the respective dates in the five cities named.

	1888	1904
Boston	16.6	16.2
Chicago	9.3	18.6
Louisville	16.7	17.2
New York	26.2	12.
St. Louis	19.3	15.2

It is difficult to account for Chicago's devoting only 9.3% of school time to arithmetic in 1888 and then having increased to exactly twice that amount in 1904; probably the New York change from 26.2% to 12% was connected with the movement toward enriching the course of study. Certain it is that the outcry against the so-called fads arose soon after 1904.

The importance of knowing the effect of such differences in time expenditure was one of the incentives to the present study. It would be interesting to state just here the relative standings of the Chicago and New York schools, and without any breach

[1] *Public Elementary School Curricula*—Ph.D. Dissertation, Columbia '05.

of confidence it may be stated that the comparatively small number of schools tested in each of these cities are now giving about the same amount of time—a medium amount—to arithmetic and the scores of the schools of both systems are among the thirteen highest.

Much help has been afforded by Professor Strayer's investigation of the time allotments of thirty American cities selected at random.[1] Professor Strayer found that the amount of time devoted to arithmetic varies from 2485 week minutes[2] for the eight grades in Cambridge, Massachusetts to 1095 week minutes in Watertown, Massachusetts. Unfortunately, neither of these cities was tested, but Montclair, New Jersey, which was found to give only 1275 week minutes, was tested and it stood among the very best.

A very comprehensive survey of the arithmetic work of the first three grades is that of the Massachusetts Superintendents' Association. The report is signed for the committee by Mr. F. E. Parlin, Superintendent of Quincy, Massachusetts, and is based on the answers of the superintendents of seventy-four cities. It concludes as follows:

"The schools which teach number in the first grade give much more time to the subject even in the second and third grades than do the schools which begin work in those grades. Unless it can be shown that the pupils of the former schools are better arithmeticians in the upper grades, or in some way superior to the others, it would seem that those schools are doing a large amount of needless, if not harmful work, and are wasting much valuable time and material. Personally we wish the same series of tests might be given to the pupils in the grammar grades of several schools representing each extreme, in order that a comparative study might be made of the results."

It is understood that, with the authority of the Superintendents' Association, the committee is planning to make such tests as soon as the practice of omitting arithmetic from the first two grades has been in operation long enough to affect the grammar grades. The present study was able to reach several systems whose 6A pupils had come up with a small amount of time

[1] As yet unpublished.
[2] This number is computed by finding the sum of the time devoted to arithmetic for one week in each of the grades.

devoted to arithmetic in the first two grades. The achievements and time distributions are given in tables XXII, XXIII and XXIV on page 55, from which they may be compared with the achievements of systems with other distinctive procedures.[1]

Another study that is very significant in this connection is that of Miss King, *Fifth Grade Programs.*[2] Miss King finds that of one hundred seventy-five fifth grade programs, selected at random from fifth grades of several cities, sixty-six per cent place arithmetic during the first hour in the morning, twenty of the others place it during the second, i. e., 86% of the makers of these one hundred seventy-five fifth grade programs mean to place arithmetic during the best hours of the day. That the time of day makes any appreciable difference, Miss King's research tends to disprove. One thing that is certainly proved by this and previous related studies is that the time of day does not influence the results when an outsider gives the tests. This is particularly pertinent to the present study in connection with the fact that some schools were necessarily tested in the later hours of the day. Principals and teachers may rest assured that their pupils did just as well as though they had been tested during the morning hours.

When one adds to the evidence above cited (1) the fact that both Payne and Strayer found arithmetic getting a larger percentage of school time than any other subject, unless some of the phases of English be counted as a single subject, (2) the fact that of the thirty-nine principals who filled the last part of the time cost blank[3] thirteen of them estimated the teacher's preparation time for arithmetic as greater than for any other subject, (3) that for the twenty-one systems that assign work for preparation outside of school hours the average of the estimates for arithmetic is two hundred and seven week minutes,[4] while the average of the estimates for *all other* subjects is only six hundred forty-three week minutes,[4]—with these facts in mind, one can not doubt that arithmetic receives the 'lion's share' of time and energy in at least some systems. Whether it flourishes in

[1] See page 62 for brief reading of table XXIV and conclusion.
[2] *M. A Thesis*—Teachers College, Columbia University—published in Teachers Bulletin University of Cincinnati Press.
[3] See page 50.
[4] Original blanks and summary on file at Teachers College.

proportion to its opportunities is the subject of the following pages.

The Data Blank and Its Uses

The following is a copy of the blank submitted to each principal at the time of the tests.

HELPFUL DATA

(Furnished by the Principal at time of test)

Date
City
Name or number of school
Name of Principal

N. B. The main value of this data is the help it will give in determining how the time and energy of pupils and teachers have been used in representative systems of schools. In order to have your system accurately represented, please let your figures be exact wherever possible, but please fill out everything called for, and if you are in doubt as to the accuracy, place the following letters after the doubtful figures: A, if nearly certain of correctness; B, if less certain; C, if pure guess.

Time present Grade VI used in Arithmetic.

1. Reciting. State number and length of periods per week.

Before	Grade I.		
In	" II.	In Grade	IV.
"	" II.	" "	V.
"	" III.	" "	VI.

2. Studying.

(a) In school hours. State number and length of periods per week while in

Grade I.	Grade IV.
" II.	" V.
" III.	" VI.

N. B. If teacher probably helped children during study periods, state about what part of periods were so used.

(b) Outside of school hours. State average daily time used by pupils while in

Grade I.	Grade IV.
" II.	" V.
" III.	" VI.

Time present Grade VI used in the study of all other subjects outside of school hours. State average daily time used by pupils in

Grade I.	Grade IV.
" II.	" V.
" III.	" VI.

Approximate average weekly time class teachers of present Grade VI have used outside of school hours on the different subjects,—planning work, correcting papers, etc.

Grade	I	II	III	IV	V	VI
Arithmetic....	....	....	....	....	....	....
Reading.......	....	....	....	....	....	....
Language	....	....	....	....	....	....
Geography.....	....	....	....	....	....	....
History.......	....	....	....	....	....	....
Manual Work..	....	....	....	....	....	....
Art Work	....	....	....	....	....	....
Nature Study..	....	....	....	....	....	....
Etc...........	....	....	....	....	....	....

Sample copies of the following would help very much: tests given Grade VI in this and previous years; plan books made by previous and present teachers; lesson assignments given in present and previous years.

Course of study taught present Grade VI. If current course not followed, note the principal differences. A copy of old course with statement of how present Grade VI followed it would help most.

The main use of these blanks was to furnish the time expenditure as measured by the time that the pupils tested devoted to arithmetic. An estimate of time "used in the study of all other subjects outside of schools hours" was asked for so as to have a basis for comparison. The approximate preparation time of teachers was asked for, not with any hope of getting estimates that would be reliable as to *exact* amounts of time, but with the idea of getting judgments of the *relative* amounts of time and energy as divided among the subjects. The question on which some help was hoped for from this part of the blank is, *to what extent does arithmetic get the 'lion's share' of teachers' time and energy.*

It is hoped that these blanks were saved from the usual vagaries of questionnaires (1) by being personally presented by the author, (2) by the personal interest of those answering, and (3) by allowing for gradations in accuracy of answers.[1] A majority of the principals filled the blanks while the tests were being given, the author having previously explained their nature and purpose. All were sufficiently interested in the study to fill all the more important parts of the blank and, with a very few exceptions, all public school principals filled the *entire* blank. Some few public school principals and most others professed not to be able to even guess at the teacher's preparation time. As

[1] See note marked N. B. at top of blank.

a too definite explanation of this part of the inquiry would have defeated its purpose by making some over-conscious in their judgments, it was not thought best to say that the amounts of time assigned made *no difference* so long as the ratio of arithmetic to the other subjects was maintained. Hence some principals declined to give help in this part of the study.

As the wording of the blank indicates, the time asked for was that *spent by the pupils that were tested* when they were in the respective grades; and the measure used for a system was the average of the amounts furnished by each of the principals of that system.[1] Taking the entire twenty-six systems, the number of principal's blanks to a system varies from one to six; and for the seventeen public school systems, from two to six; for nine of the seventeen there were four or more and for six of the remaining eight there were three.

TABLE XXI[2]

Amounts and Variabilities of Time Expenditures in Twenty-six Systems.

Systems in order of Time Expenditure	Week Minutes	Deviation from Median	Deviation in % of Median
XXII......................	507	−642	−56
XXV......................	*340* 722	−427	−37
XXVI......................	*150* 837	−312	−27
XXI......................	*300* 865	−284	−25
X......................	*81* 921	−228	−19
III......................	944	−205	−18
XXIV......................	950	−199	−17
V......................	*362* 971	−178	−15

[1] In the case of the sixth grade the amounts furnished by teachers was also included in the average.

[2] As stated in Part II, the tables are their own best interpretation and the brief notes that accompany them are not meant to do more than facilitate a general survey by giving sample readings.

TABLE XXI—*Continued*

Systems in order of Time Expenditure	Week Minutes	Deviation from Median	Deviation in % of Median
I	*175* 1068	−81	−7
VI	1126	−23	−2
XVI	*389* 1127	−22	−1
XI	*76* 1130	−19	−1
XII	*288* 1148	−1	−.08
XXIII	1150	1	.08
XX	*450* 1161	12	1
XV	1173	24	2
II	*72* 1247	98	8
VIII	*552* 1258	109	9
XVIII	*90* 1265	116	10
XIX	*319* 1276	127	11
XVII	*84* 1283	134	11
IX	*620* 1559	410	34
XIV	1560	411	34
VII	1573	424	37
XIII	1626	477	42
IV	1854	705	62

Reading the column of table XXI marked *week minutes* the lower numbers opposite each system give the school time expended in studying and reciting arithmetic for one week of each of the first six grades; the upper, the estimated study time used out of school. As the *school time* expenditures are obviously the more reliable, they are the measure of time cost used in this table and elsewhere, except as otherwise specified. The column marked *deviation from the median* is self-explanatory, the median being the measure above and below which half the cases lie, which in this table is 1149. The third column is the deviations in per cent of the median. These three columns each show the wide variability as to time expenditure among the systems. The showing of the first column may be briefly summarized by noting that the number of week minutes varies from five hundred seven for system XXII to one thousand eight hundred fifty-four for system IV, with an average deviation of two hundred twenty-two. That is, system IV spends over three and one-half times as much time on arithmetic as system XXII, and the other systems range between these extremes. The wide variation comes out even more strikingly in the second and third columns.

This table furnishes the basis for comparing time expenditure with abilities produced, as shown in the following tables. The next two tables begin this comparison.

Table XXII | **Table XXIII** | **Table XXIV**

Systems	Comparative Achievements: Average serial standing	Comparative Achievements: Serial standing in reasoning	Comparative Achievements: Serial standing in fundamentals	Comparative Time Expenditure: Serial standing in time expenditure	Comparative Time Expenditure: Week minutes devoted to arithmetic	Comparative Time Expenditure: Week minutes devoted to all subjects	Comparative Time Expenditure: % of time to arithmetic	Time Distribution Among Grades (Lower numbers show week minutes devoted to arithmetic; upper show % of school time devoted to arithmetic in each grade): I	II	III	IV	V	VI
XXIII	1	1	1	14	1150	9675	12	*7* 100	*6* 100	*12* 200	*15* 250	*15* 250	*15* 250
XXV	3	4	2	2	722	8700	8	....	*7* 100	*9* 140	*10* 155	*9* 130	*13* 197
XXII	4½	5	4	1	507	7200	7	....	*8* 90	*8* 90	*8* 90	*7* 90	*11* 147
XX	5	7	3	15	1161	8200	14	*7* 80	*10* 113	*12* 210	*14* 240	*15* 265	*15* 253
XVII	7½	3	12	21	1283	7500	17	*2* 27	*12* 158	*20* 250	*20* 258	*24* 300	*23* 290
VIII	8	11	5	18	1258	9600	13	*2* 25	*14* 233	*15* 250	*15* 250	*15* 250	*15* 250
XV	8	9	7	16	1173	8025	15	....	*11* 147	*15* 213	*20* 292	*17* 250	*18* 271
III	9	10	8	6	944	8025	12	*10* 125	*9* 125	*11* 150	*11* 150	*12* 165	*16* 229
XXIV	10	2	18	7	950	8775	11	....	....	*14* 200	*17* 250	*17* 250	*17* 250
X	10	14	6	5	921	8550	11	....	*7* 88	*12* 154	*12* 184	*14* 216	*18* 279
I	11	13	9	9	1068	9375	11	....	*9* 130	*13* 213	*14* 238	*14* 238	*15* 249
IV	12	4	20	26	1854	8400	22	*28* 249	*20* 300	*20* 306	*24* 361	*20* 300	*23* 338
II	13	15	11	17	1247	9900	13	*8* 121	*12* 192	*14* 217	*14* 225	*15* 233	*16* 259
XXI	13	16	10	4	865	7650	11	*8* 80	*10* 100	*10* 100	*12* 180	*14* 210	*13* 195

Table XXII—Con. | **Table XXIII—Con.** | **Table XXIV—Con.**

Systems	Comparative Achievements			Comparative Time Expenditure				Time Distribution Among Grades (Lower numbers show week minutes devoted to arithmetic; upper show % of school time devoted to arithmetic in each grade)					
	Average serial standing	Serial standing in reasoning	Serial standing in fundamentals	Serial standing in time expenditure	Week minutes devoted to arithmetic	Week minutes devoted to all subjects	% of time to arithmetic	I	II	III	IV	V	VI
VI	13	12	14	11	1126	9000	13	....	8 127	12 177	18 266	18 266	19 290
XVI	14½	6	23	12	1127	9000	13	5 75	8 113	13 187	18 263	17 251	16 238
XIII	15	17	13	25	1626	8475	19	....	26 388	23 350	18 288	19 300	19 300
XVIII	16	8	24	19	1265	8700	15	6 75	5 75	15 225	20 300	20 300	19 290
IX	17½	19	16	22	1559	9000	17	13 200	15 225	18 275	18 275	18 275	21 309
XI	18½	22	15	10	1130	8575	13	....	11 157	15 216	18 250	18 250	18 257
XIV	18½	18	19	23	1560	8850	18	15 225	16 245	18 270	19 280	19 270	19 270
XII	19	21	17	13	1148	8400	14	....	6 81	18 226	16 255	19 288	19 298
XXVI	22½	23	22	3	837	7200	12	7 80	10 125	10 125	13 150	13 150	17 207
VII	22½	20	25	24	1573	7800	20	13 175	19 262	22 300	22 300	22 300	17 236
V	23	25	21	8	971	8700	11	8 113	10 154	11 167	12 175	13 183	13 179
XIX	25	24	26	20	1276	9000	14	8 125	10 150	17 250	17 250	13 200	20 301

...... =No time assigned.

As seen from its heading table XXII gives the systems in order of achievements. These serial standings are derived from tables III and IV. Reading from the top, system XXIII has an average serial standing of *one,* being lowest in both reasoning and fundamentals; system XXV ranks *three* in average serial standing, being *fourth* from lowest in reasoning, and *second* from lowest in fundamentals; the readings for the other systems are similar.

Tables XXIII and XXIV keep the same order of systems and show the time expenditure. The first line of table XXIII reads, —system XXIII ranks fourteenth from the lowest in time expenditure with 1150 week minutes devoted to arithmetic, 9675 week minutes devoted to all subjects, the 1150 week minutes devoted to arithmetic being 12% of the 9675 week minutes devoted to all subjects. Similarly for the other systems, e. g., system XXV with a serial standing in abilities of *three,* and a serial standing in time expenditure of *two,* spends 722 week minutes on arithmetic, and 8700 week minutes on all subjects, arithmetic costing 8% of all the school time. The reader will recognize that the third column, which gives the time devoted to all subjects for one week of each of the first six years, is the only new data of this table, column two being the same as given in table XXI and the first and fourth columns being derived from the others.

Probably the first essential shown by this table is the lack of correspondence between the serial standing in time cost and the serial standing in abilities, e. g., the system with the lowest time cost is found by referring to table XIII, to be system XXII which is seen in table XXII to rank *four and one-half* in average abilities. Similarly, the system that ranks *fifteenth* in time cost, ranks *fifth* in abilities, etc. Another noticeable showing is the wide variability in the *school time* of the systems. It will be seen to vary from 7200 to 9900 week minutes. This time includes recesses and it means that lengths of school days vary from an average of four hours to five and one-half hours. And if the names of the systems were given, it would be recognized that almost invariably the longer school hours are accompanied by the least amount of variation in program, such as physical education, field trips, assemblies, etc. Perhaps the other most

striking fact of this table is the wide variation in the per cent of time devoted to arithmetic. It varies from 22% for system IV to 7% for system XXII, a difference of more than three to one.

As table XXIV is part of the discussion of factors in time expenditure, its sample readings are given under that heading, page 62.

The Relation of Time Expenditure to Abilities Produced

The reader found one indication of the relation, or lack of relation, between time cost and products in tables XXII and XXIII. Each of the three following tables expresses these same facts.

Table XXV

Comparison of the achievements of the systems having less than median time cost with those having more.

	Combined Scores of the Thirteen Systems			
	With *less* than median time cost	With *more* than median time cost	With *less* than median time cost	With *more* than median time cost
	Without home study		Including home study	
Reasoning.....	7519	7893	7277	8135
Fundamentals..	40751	40273	37165	43859

The above details are compiled from the scores of individual systems as given in tables III and IV, the time cost being that given in table XXI. As measured by the time used in school the thirteen systems with less than the median time cost stand slightly the better; and as measured by the time including home study, the thirteen systems with more than the median time cost stand somewhat the better. The time used in school is doubtless the more exact measure, but as shown in table XXI some systems depend on home study to a considerable extent. Hence both measures are used. The lack of relation indicated in this general way is shown more accurately in the following table in terms of coefficients of correlation.

TABLE XXVI

Correlation of Time Expenditures with Abilities.

		Pearson	Median ratio	Cosine $\pi\omega$	Average[1]
Without Home Study	Reasoning and Time Expenditure........	.110	−.010	−.125	−.008
	Fundamentals and Time Expenditure........	.412	−.030	−.125	.086
Including Home Study	Reasoning and Time Expenditure........	.214	.055	.125	.131
	Fundamentals and Time Expenditure........	.361	.557	.562	.493

Average of the four averages .176
See tables III, IV and XXI for entire distributions.

As might have been expected from table XXV, the correlation is practically *zero* without home study and not very much above zero including home study. As anything less than .25 indicates little relationship and the average of the averages of these coefficients is only .176, there is little relationship indicated between the time expended by these twenty-six systems and the abilities produced. This fact is again expressed in the next table. In passing, it is worthy of note that all the coefficients of time with reasoning are lower than those of time with fundamentals. Just why this is and what it means is not yet clear, but it probably has significance for teaching these phases of the subject.

[1] As stated in connection with the coefficients of Part II, the average of the coefficients derived by the three methods is regarded as the best single measure.

TABLE XXVII

Ratio of Time Expenditures to Abilities.[1]

Systems	Average Ratios		Reasoning ratios		Fundamental ratios	
	Serial standing of systems	Time cost to reasoning and to fundamentals	Serial standing of systems	Time cost to reasoning	Serial standing of systems	Time cost to fundamentals
IV	1	2.26	1	3.99	4	.520
XXIII	2	1.92	2	3.22	1	.624
XVII	3	1.65	3	2.88	7	.421
XIII	4	1.54	4	2.55	3	.533
XX	5	1.45	7	2.36	2	.535
XVIII	6	1.41	5	2.48	13	.336
XIV	7	1.40	7	2.36	6	.438
VIII	8	1.39	8	2.33	5	.457
IX	9	1.353	9	2.25	5	.457
XVI	10	1.352	6	2.40	18	.304
XV	11	1.31	11	2.20	8	.422
VII	12	1.28	12	2.14	9	.415
XXIV	13	1.24	10	2.21	21	.270
II	14	1.22	14	2.02	7	.421
VI	15	1.20	13	2.04	11	.354
I	16	1.15	15	1.93	10	.363
III	17	1.05	16	1.77	16	.331
XII	18	.943	17	1.55	13	.336
XXV	19	.941	17	1.55	15	.333
X	20	.93	18	1.53	14	.335
XI	21	.913	20	1.48	12	.346
XIX	22	.91	19	1.50	17	.311
XXI	23	.83	21	1.37	19	.293
V	24	.67	23	1.06	20	.272
XXII	24	.65	22	1.08	23	.219
XXVI	26	.64	24	1.05	22	.227

[1]For the data from which these calculations were made, see first column of table XXI, page 52 and the first columns of tables III and IV page 21. The absence of known zero points makes such computations inadvisable except in connection with the more reliable evidence of the preceding table

The order of systems in this table is determined by the first column which gives the average serial standing as determined by the ratios of time to products. The right hand column under each heading gives the ratio of time expenditure to abilities produced, and the left hand column gives the serial order of that system as measured by the highness of the ratio, i. e., highness of cost per unit of product, e. g., in system IV the ratio of time to reasoning is 3.99 (see fourth column), the highest ratio in reasoning (determined by dividing the time cost, 1854 week minutes, by 464, the points made in reasoning). The ratio of time to fundamentals in this system is .52; giving an average ratio of 2.26. That is to say the ratio of time to abilities in system IV is as 2.26 to 1, the highest among the twenty-six systems.

That there is no direct ratio between time expenditure and abilities is again shown by this table. For example, system XXII, which spends the least amount of time (see table XXI), ranks fourth from the lowest in abilities (see table XXII), ranks 25th, that is next to the highest, in ratio of time cost to abilities produced; and what is even more striking, system XXVI, which spends third from the least amount of time ranks third from the highest in abilities and 26th or *highest* in the ratio of time cost to abilities produced.

That a large amount of time expended is no guarantee of a high standard of abilities may again be convincingly seen by comparing the ratios of the five systems spending the smallest amount of time with the five spending the largest. Of the five spending the least time, the average ratio is .80, which corresponds with the 23d or the 3d from the best in ratio; and of the five spending the greatest amount of time, the average ratio is 1.57 which corresponds with the fourth poorest in ratio.

The last three tables have each shown the decided lack of relationship between time cost and abilities produced, and hence for these systems it is evident that there is practically no relation between time expenditure and arithmetical abilities; and, in view of the representative nature of these twenty-six systems, it is probable that this lack of relationship is the rule the country over.

This is not to say that a certain amount of time is not essential to the production of arithmetical abilities; nor that, given the

same other factors, operating equally well, the product will not increase somewhat with an increased time expenditure. What is claimed is that as present practice goes, a large amount of time spent on arithmetic is no guarantee of a high degree of efficiency. If one were to choose at random among the schools with more than the median time given to arithmetic, the chances are about equal that he would get a school with an inferior product; and conversely, if one were to choose among the schools with less than the median time cost, the chances are about equal that he would get a school with a superior product in arithmetic.[1]

So far, then, as ability in arithmetic means ability to handle such foundation work as is measured by the tests in this study, this 'essential' has not *necessarily* suffered by the introduction of other subjects and the consequent reduction of its time allotment.

The lack of correlation between time used and abilities produced is doubtless attributable to various reasons. An attempt is made to account for the achievements of the respective systems as part of the summary, page 69. Excluding, for the present, the other influences that are probably potent in producing abilities, the remainder of this part of the study is given to considering some means of economizing time.

Some Factors in Time Expenditure

With a given amount of time, what is the most economical use to make of it? The answer which the more successful systems have made to this question is what has enabled them to do good work in arithmetic and still have time to devote to enriching their school life.

The Distribution of Time Among Grades

The various apportionments of time to grades as found among the twenty-six systems are shown in table XXIV on page 55. This table reads across the page from tables XXII and XXIII; and shows the time distribution among grades for each of the systems, the lower numbers being the week minutes devoted to arithmetic in each of the grades; and the upper, the per cent

[1] And it is the opinion of the author that the chances are much better that one would get a school with a superior product in *education*.

that the arithmetic time is of the entire school time of that grade.

These distributions are indices of distinctive educational procedures, expressed in terms of administration. The fact that systems vary from no time for arithmetic in grade one to two hundred forty-nine week minutes must indicate radically different ideas of teaching arithmetic. That in ten of these systems the pupils of grade one are not taught arithmetic at all, while in others they are taught it for from two to twenty-eight per cent of all the time, is a striking indication of the wide diversity of opinion held by those who construct courses of study and determine time allotments. It is also notable that the systems differ in that a majority begin with a comparatively small amount of time and gradually increase it, while a few have practically the same amount from the first grade.

The relative advantages of these respective amounts and distributions of time are at present unknown. Each superintendent is standing for what he thinks or hopes is best.

Laborious as has been the gathering of the data, the author must frankly acknowledge that for this purpose it is still inadequate for reliable conclusions; and while *a priori* the author believes that the lessening of time devoted to the formal aspects of arithmetic, and the enriching of the experience of the pupils in the lower grades is a move in the right direction, the results of this study do not show conclusive evidence in support of this belief. As suggestive evidence that such a procedure can produce good results, reference is made to systems V, XXVI and XII, which as shown in tables XXI and XXII stand second, third and fourth from the best in abilities.

A recent tendency is to diminish the time devoted to arithmetic in the first two grades. An increasing number of superintendents over the country claim that *no time* is devoted to arithmetic in the first two years; but when the principals and teachers are consulted one usually finds that some time is used in the second if not in the first grade. While the author understood from the superintendents of several of the systems tested that no time was given to arithmetic in grades I and II when the pupils tested were passing through those grades, table XXIV shows only *one* system in which the principals reported no time cost for these two grades. There are ten with no time

cost in grade I and a majority with less in grade II than in the grades above.

In order to help determine which distribution of time yields the best results, i. e., whether it is best to spend time equally in all grades or concentrate it in the grades above the second the following steps were taken:

1. The per cents of arithmetic time spent by each system in grades I and II were computed from tables XXIII and XXIV. This gave a range of 0% to 30%, with the median 17.6%, i. e., one system distributed none of its arithmetic time to the first two grades, thirteen systems only 0% to 17%, and thirteen systems 18% to 30%.

2. The gross achievements of the thirteen systems spending more than the median per cent of time in grades I and II were compared with the gross achievements of the thirteen systems spending less than the median per cent of time in grades I and II. This gave a gross score of 8520 in reasoning and 42521 in fundamentals for the first thirteen systems and 6892 in reasoning and 38503 in fundamentals for the second thirteen, showing that the thirteen systems that tended to distribute their time most equally among the grades did about 10% better in fundamentals and 25% better in reasoning.

3. Another way of testing the efficacy of spending a small amount of time on arithmetic in grades I and II is to take the thirteen systems that stood best in the ratio of time to products and see how they distributed their time. This was done using table XXVII. Of these thirteen systems six were among the thirteen which spent less than the median per cent of time on arithmetic in grades I and II and seven were among the thirteen spending more than the median per cent of time on arithmetic in those grades.

If it were shown that those systems that concentrated in the grades above the second secured a better return for their expenditure, the argument that it is more economical to so concentrate would be sustained but these twenty-six systems do not show this to be the fact.

To those who believe in concentrating in the grades beyond the second, the above findings will not be convincing and the question is certainly worthy of further investigation. On the basis of the present study the best distribution seems to be that

which assigns not less than 20% of the arithmetic time to grades I and II. Judging from the results among these twenty-six systems there is arithmetic work that pupils can profitably do before reaching grade III. But how much time ought to be spent on arithmetic in grades I and II will doubtless be influenced by at least the following factors,—(1) the other uses that can be made of the school time of those grades, and (2) the number of grades above the second in which the pupils will probably be in school to get arithmetic work.

The Use of Time Within a Grade

After a certain amount of time has been apportioned to a grade, the teacher and the supervisor are confronted with the question of how best to use it; and conversely, the best use of time will very likely lessen the amount needed.

Probably one of the most potent factors in the economy of time is suiting the teaching to the needs of those taught. As was shown in Part II, there is a wide variability among individual pupils as to abilities and the relation between the various abilities is surprisingly slight. From this it follows that the needs of the usual group vary widely. How to meet these individual needs economically is an important question, with bearings on grading, sectioning, individual instruction, etc. One fact seems established, viz., that the wide variability makes the "lock step" of formal grading and promotion not only inhuman but uneconomical; and logically it makes it quite as uneconomical to expect the same response to the same kind of teaching in one phase of arithmetic as in another. Economy then would demand that the teacher and supervisor know the respective abilities of the different pupils in each phase of the subject that they may suit the teaching to the needs and capacities. But this does not mean that the method of individual instruction, used extensively, is the most economical; for according to the usual distribution of mental traits "over two-thirds of the individuals of any homogeneous group are centered within the middle third or less of the total range of ability;"[1] and an examination of tables X and XI, pages 30 and 31, shows this to be true of the individual pupils of the four systems selected at random.

[1] Thorndike's *Educational Psychology*, p. 22.

Economical grouping must take this into account, e. g., if sixty pupils are to be placed in three groups, they will not be divided into three equal sections of twenty each but more nearly in two sections of ten each and one of forty.

That difference in economy of time does effect results is proven in the wide difference among systems as to the number of problems attempted. Only about 38% of all the pupils in the five systems securing the least number of scores attempted problem seven, while about 86% of the entire number in the five highest systems attempted it.

This fact is most fully shown by comparing tables III and IV page 21, with tables VI and VII, page 24. The most significant single figures are the coefficients of variability, table XII, page 33. From these tables it is seen that with plenty of time the differences between the standings of systems is very much reduced, the coefficients of variability being only .12 for fundamentals, and .07 for reasoning when the scores from only the first six problems are counted, as against .20 for fundamentals and .13 for reasoning, when the scores are counted for all the problems attempted. After allowing for the increased difficulty of the problems, these differences in variability show that with economy of time much more could have been accomplished by the systems with lower scores, and doubtless some more could have been accomplished by certain of those standing higher.

What then is done with the time? Besides a probable uneconomical grouping, several sources of waste were noted as the author observed the work of the different systems, viz., lack of expedition in handling materials, changing work, etc.; unnecessary ruling; excess of writing; unnecessary indicating of steps; excess of labelling; useless indicating of processes, e. g., easy cancellation, etc. The papers show that most of the systems standing low and several of those standing high could do better with economy along these lines.

In the parallel columns below, this point is illustrated in the work for a single problem, number two of the reasoning test. Here are copies of the ruling, figures, labelling, etc., exactly as found on pupils' papers. In the left hand column are a few typical illustrations of unnecessary use of time; in the right, a few types of economy of time:

2.

$$\begin{array}{r|l} & \text{5 papers} \\ \hline \$\ .02 & 10 \end{array}$$

2.

5 papers

2. 5c cost of 1 S. E. Post.
4
—
20c " " 4 " " "
½ of 20c = 10c.
10c he kept.
10c he had left for papers.
2c cost of 1 paper.
2c: 10c had for papers.

5 times.
He bought 5 papers.

2.

$$\begin{array}{r|r} & \$\ .05 \\ & 4 \\ \hline 2 & \$\ .20 \\ \hline \$\ .02 & \$\ .10 \\ \hline & 5 \text{ Ans.} \end{array}$$

(2).

First Step: Find how much John received for the Saturday Evening Posts by multiplying 5 cents by four.

$$\begin{array}{r} \$\ .05 \\ 4 \\ \hline \$\ .20 \end{array}$$

Second Step: Find how much money he kept by dividing \$ .20 by 2.

$$\$\ .20 \div 2 = \$\ .10.$$

Third Step: Find how many Sunday papers he bought by dividing \$ .10 by \$ .02.

$$\$\ .10 \div \$\ .02 = 5 \text{ papers.}$$

2.

$$\begin{array}{r} 4 \\ 5 \\ \hline 20 \end{array} \qquad 2\overline{)10} \quad 5 \text{ Ans.}$$

The last one in the left column and its parallel in the right column come from the work of pupils in the same system, and they are typical of the uses made of arithmetic time in this system. What could be more emphatic evidence of the need of supervision?[1] When one notes the difference in time cost for this one problem as part of one fifteen minute exercise and considers what it would mean in entire lesson periods day after day, the waste is quite appalling.

To the plea for allowing individuality among pupils the reply is that the variety of such forms as are illustrated in the right hand column allows sufficient opportunity for individuality and that furthermore pupils would hardly go into the excessive detail illustrated in the left hand column unless taught to do so.

To the claim that more pupils would have economized time if they had been directed to put down answers only or work only, the reply is,—the same directions were given to all pupils, i. e., no directions were given except those printed at the top of the test papers. It is believed that this affords a fair basis on which to judge the proficiency of boys and girls in the solution of such everyday problems as the first ten of the test. One purpose of the test was to measure what pupils did when they were not told the best way, i. e., to measure how they would economize time, etc., without the teacher standing by to say do *thus* and *so*,—to measure how far they were prepared to solve the problems in life.

And however fruitful the use of time for a somewhat extensive analysis of such simple problems may be at certain stages, it certainly seems reasonable that high sixth pupils should have passed beyond these stages and have had sufficient practice to have fixed the habits of economical solutions.

Another waste of time that was noted is counting. Among some systems it was quite prevalent, varying from saying the number to using the fingers, or dots. While such practices may be permissible in the lower grades as aids to accuracy, they certainly should be replaced by more serviceable habits by the time pupils reach high sixth.

[1] There is practically none in this system.

SUMMARY

1. In so far as these twenty-six systems are a representative measure, there is very little relation between arithmetical abilities and time expenditure in present practice. Many systems are wasting time on arithmetic. They not only do not afford a rich life to the child, but they do not afford him abilities in arithmetic.

2. A few systems are probably failing to give arithmetic its due share of time.

3. In that all the coefficients that include home study are higher than those without may indicate that home study is a factor in determining abilities; compare Rice's contention to the contrary, also Cornman's position.

4. The fact that all the coefficients for reasoning are lower than those for fundamentals may be due to one of three conditions or to a combination of all three: (a) The time devoted to fundamentals may be more fruitful, i. e., the teaching of fundamentals may be better; or (b) more of the time may be devoted to fundamentals; or (c) ability to reason may be much less dependent on amount of teaching than ability in fundamentals.

5. The best distribution of time among grades is yet to be determined. That several systems do well with a small amount in the lower grades suggests that a large amount is not essential.

6. There is much waste of the time allotted to arithmetic.

SUGGESTIONS FOR DETERMINING TIME ALLOTMENT

A rational method of arriving at a satisfactory time allotment for any particular system would involve at least the following steps:

1. Comparing the achievements attained from a reasonable time expenditure, e. g., the median time cost of these 26 systems —(about 1200 week minutes)—with the achievements attained by other systems having about the same cost.

2. Studying the influence of the various other factors that determine achievements in the systems compared.

3. Making sure that lack of ability is not due to some other than the time factor.

4. Gradually adjusting the time expenditure to produce the desired results—presumably not less than the systems with the better records, e. g., systems XXVI, V, XI, or XII, whose scores in reasoning are 700 to 900 points per 100 pupils; and in fundamentals 3500 to 4000 per 100 pupils—as measured by the tests employed in this investigation.

PART IV.—ARITHMETICAL ABILITIES AND COURSES OF STUDY

The Problem

"What shall be the course of study?" is a question that is receiving much attention. The curriculum has definitely come to be regarded as an index of educational progress. A common question regarding a school is, "What is its course of study?" And what ought to be the course of study has been a storm center round which many a controversy of opinion has been waged. This part of the present study is an attempt to determine how far the division of the elementary school curriculum called arithmetic is functioning as a factor in producing abilities. The problem may be stated: *What is the relation between arithmetical abilities and the course of study in arithmetic?*

Steps in the Solution of the Problem

(1) The first step toward the solution of this problem was the securing of the courses of study that were in the hands of the teachers who taught the pupils that were tested. After stating the need in a personal interview, it was formulated in a note of which the following is a copy. Then after receiving the material the second note, below, was sent to make sure that the course of study material at hand was that according to which the pupils were taught.

Note Stating Need for the Course of Study Followed by the Teachers of Pupils Tested

Dear Sir:—

I am glad to write of progress on the Arithmetic study, the results of which I hope to share with you.

As part of the study I am planning to use the Course of Study followed by the teachers who taught the pupils that I tested. Could you furnish me with a copy? If the statement in the present course is different from that used in the lower grades when the children tested were in those grades, a copy of the older course or courses is needed as well as the present course.

Thanking you again for your past interest and help, and trusting that the study may have the benefit of these Courses of Study at your early convenience, I am, Very respectfully,

Note Giving Opportunity to Correct Course of Study Materials to be Used

My dear—

I am glad to write you that my study of arithmetic, in which you so kindly co-operated, is progressing and promises to be available ere long, when I shall be pleased to send you a copy. No names will be used in discussing the relative excellences of systems, but I shall retain a key by which I can enable any one to locate his system among the twenty-six studied.

As part of the research is based on the Courses of Study followed by the teachers who taught the pupils I tested, I am anxious to be certain that what I have is accurate and adequate for your system.

At present I have the benefit of

If there is any other Course of Study or Syllabus material that I should have for your system, will you kindly let me know at your early convenience. Very sincerely,

(2) The second step was the securing of ratings of the courses of study. This was done through the co-operation of twenty-one Professors and graduate students of education, each of whom had had practical experience with and made a study of elementary school curricula. Each of these scorers was furnished with a copy of the following directions and there is every reason

to believe that all gave intelligent and unbiased judgments. The results appear in the table following the directions.

Concerning the Rating of Courses of Study

Judges please read before scoring

I. Some Factors Determining Relative Excellence.

(N. B. The following enumeration is meant to be suggestive rather than complete or exclusive. And each scorer is urged to rely primarily on his own judgment.)

1. Helpfulness to the teacher in teaching the subject-matter outlined.
2. Social value or *concreteness* of sources of problems.
3. The arrangement of subject-matter.
4. The provision made for adequate drill.
5. A reasonable minimum requirement with suggestions for valuable additional work.
6. The relative values of any predominating so-called methods—such as Speer, Grube, etc.
7. The place of oral or so-called mental arithmetic.
8. The merit of text-book references.

II. Cautions and Directions.

(Judges please follow as implicitly as possible.)

1. Include references to text-books as parts of the Course of Study. This necessitates judging the parts of the texts referred to.
2. As far as possible become equally familiar with all courses before scoring any.
3. When you are ready to begin to score, (1) arrange in serial order according to excellence, (2) starting with the middle one score it 50, then score above and below fifty according as courses are better or poorer, indicating relative differences in excellence by relative differences in scores, i.e. in so far as you find that the courses differ by about equal steps, score those better than the middle one 51, 52, etc., and those poorer 49, 48, etc., but if you find that the courses differ by unequal steps show these inequalities by omitting numbers.
4. Write ratings on the slip of paper attached to each course.

TABLE XXVIII

SCORES FOR COURSES OF STUDY OF THE TWENTY-SIX SYSTEMS TESTED

Systems in order of excellence	Median of 19 scores	A D for 19 scores	Systems in order of excellence	Median of 19 scores	A D for 19 scores
I	32	5.7	XVII	51	4.5
V	37	4.2	XXIV	51	8.3
II	38	5.6	XV	52	9.1
X	38	7.1	XXIII	52	6.4
IV	39	5.3	XX	54	5.4
VIII	43	4.4	VI	56	6.2
VII	43	4.7	IX	56	8
XVI	45	7.1	XIX	57	7.5
XXVI	46	6.6	XXI	58	6.4
III	46	7.9	XXV	58	5.8
XIII	47	5.4	XII	60	7.9
XVIII	49	6	XXII	61	7
XIV	50	5.4	XI	65	6.3

(3) The third step in answering the question concerning the relation between arithmetical abilities and courses of study was the determining of the relation between excellence of courses and excellence of results. This relation is shown in each of the two tables below.

TABLE XXIX

This table gives the sum of the scores of those systems ranking lower than the median and those ranking higher in general excellence of Course of Study.

	Systems ranking lower than the median	Systems ranking higher than the median
Reasoning	8017	7395
Fundamentals	42905	38119

This table shows clearly that taking all the systems ranking as the poorer half in course of study, they did even better in gross score than did those ranking as the better half. This lack of relation is expressed more precisely in the next table.

TABLE XXX

Coefficients of Correlation: Course of Study ranking with Abilities—all 26 systems.

	Cosine $\pi\omega$	Median ratio	Pearson	Average
General Excellence with Reasoning..............	-.353	-.112	-.016	-.16
General Excellence with Fundamentals	-.125	-.093	-.038	-.09

That these coefficients are not only *zero* but slightly negative is convincing evidence that on the basis of the above scoring of courses of study, and the achievements of pupils as measured by the tests of the study, there is among these twenty-six systems as a group no relation between abilities and excellence of courses of study.

This is not to say that the course of study may not be a factor in producing abilities. It doubtless is a factor and as shown on page 88, it is functioning well in certain of the systems tested, but what is claimed from the above showing, is that systems vary so widely in the uses they are making of courses of study that the chances are about even that if one were to choose a system of schools with a good standing in abilities, that system would rank among the poorer systems as to course of study.

For the benefit of those who may be skeptical as to the validity of the course of study ratings, it may be said, that they were scored by what are probably the best procurable judges. That the judges worked intelligently is shown by, (1) the small average deviation among their marks, and by (2) the fact that the median of their combined marks falls at 50.5. By reference to the directions above, it will be seen that an implicit following of the directions would cause the median to fall within a range of 49.5 to 50.5.

DRILL AND CONCRETENESS IN COURSES OF STUDY

Two noticeable features of present day courses of study are emphasis on either *concreteness*[1] of subject-matter or *drill*. On

[1] For meaning of *concreteness* as used in these pages see first directions to judges under Concerning the Rating of Courses of Study as to Excellence in Concreteness, page 75.

the whole, it is safe to say that those who have been most interested in improving courses of study recently have been most concerned with improvements along the line of concreteness or intrinsic worth of the subject-matter, i. e., of the problems. The question has been raised as to whether the drill phase of the work is being properly cared for. With a view to measuring the degree to which the courses of study show that these factors are operating to produce abilities, this plan was followed; (1) the courses were rated according to the directions given below, with the results shown in tables XXXI and XXXII; (2) correlations were figured (a) between the ratings for drill and standings in abilities (b) between the ratings for concreteness and standings in abilities.

Concerning the Rating of Courses of Study as to Excellence in Concreteness of Problems

Directions to Judges

1. Rate according to the extent to which the problems are such that their solution will put the pupils in possession of the quantitative aspects of the lives they are experiencing.

2. Include references to text-books as part of the Course of Study. This necessitates judging the parts of the texts referred to.

3. As far as possible become equally familiar with all courses before scoring any.

4. When you are ready to begin to score, (1) arrange in serial order according to excellence, (2) starting with the middle one score it 50, then score above and below fifty as courses are better or poorer, indicating relative differences in excellence by relative differences in scores, i.e. in so far as you find that the courses differ by about equal steps, score those better than the middle one 51, 52, etc., and those poorer 49, 48, etc., but if you find that some courses differ by unequal steps show these inequalities by omitting numbers.

5. Write ratings on the slip of paper attached to each course.

6. Remove slips and place in envelope (found in desk).

7. Mark this envelope with your name and *Concrete*.

Concerning the Rating of Courses of Study as to Excellence in the Provision Made for Adequate Drill

Directions to Judges

1. Rate according to the extent to which provision is made for adequate drill.

(The remaining directions read the same as for *concreteness*.)

Table XXXI | Table XXXII

Ratings of courses of study as to excellence in the provision made for adequate drill			Ratings of courses of study as to excellence in the provision made for concrete problems		
Systems in order of excellence	Median of nine ratings	A. D.	Systems in order of excellence	Median of nine ratings	A. D.
I	38	9.4	IV	37	3.9
XX	40	6.8	I	37	7.3
VII	40	5	V	41	3.3
XXII	42	7.1	XVI	42	7.1
XXVI	43	8	X	42	4.2
XV	44	4.3	VIII	44.5	7.5
V	45	10	II	44	4.6
XVI	46	8.2	XXIV	44	5.3
XXIII	47	6.4	XXVI	45	8.8
VI	48	10.5	XVIII	47	4.1
IV	49	9.4	VII	48	6.3
XXI	49	8.9	XIV	49	5.5
XIV	50	9.1	XVII	50	3.9
XXIV	51	10.7	XIII	50	3.2
X	51	10.7	IX	51	7
II	52	7	III	51	6.3
XIII	54	5.1	XI	56	3.7
XVIII	55	9.7	VI	56	6.1
VIII	55	6.4	XIX	56	4
XVII	55	4.8	XV	57	6.2

TABLE XXXI—*Continued* TABLE XXXII—*Continued*

Ratings of courses of study as to excellence in the provision made for adequate drill			Ratings of courses of study as to excellence in the provision made for concrete problems		
Systems in order of excellence	Median of nine ratings	A. D.	Systems in order of excellence	Median of nine ratings	A. D.
XXV	56	6	XXIII	58	8
XII	58	11	XXI	60	10.5
XIX	58	6.2	XXV	62	2.7
III	59	7	XX	63	5.3
IX	67	8.8	XII	68	6.8
XI	68	8.4	XXII	70	3

As might be expected, the average deviation is larger in these tables and the marks are also less reliable because of the smaller number of judges. But it should also be said (1) that, with one possible exception, these nine judges are among those whose ratings varied least from the median or standard in the ratings for general excellence; and (2) that the median of the twenty-six systems for drill falls at 50.5 and for concreteness 50, showing that the scoring was done intelligently and according to the procedure stated in the directions.

Correlating these scores with those for abilities gives:

TABLE XXXIII

Cosine $\pi\omega$ Method

Excellence in provision for drill with Reasoning	.00
Excellence in provision for drill with Fundamentals	−.12
Excellence in Concreteness with Reasoning	−.23
Excellence in Concreteness with Fundamentals	−.35

These coefficients for drill show only a very little more relationship than was found between general excellence of courses of study and abilities; and for concreteness the lack of relationship is quite as evident as for general excellence.

This, again, should not be interpreted to mean that these factors were not operative in producing abilities among these sys-

6

tems nor that they are not important, for every teacher knows that they are important and the author ventures the judgment that in those systems which did well in the tests and whose courses lacked either of these elements the teachers supplied the missing element. The contention here made is that courses are not as a rule well-balanced: they run to some one phase to the exclusion of others. See samples from courses, page 79.

The Courses of the Nineteen Public School Systems

In order to test whether any particular class of schools is accountable for the lack of relationship, the following correlations were figured, using the nineteen systems that may be grouped as public schools.

TABLE XXXIV

Cosine π ω method used

General Excellence and Reasoning	−.08
General Excellence and Fundamentals	.25
Excellence in Drill and Reasoning	.25
Excellence in Drill and Fundamentals	.40
Excellence in Concreteness and Reasoning	−.08
Excellence in Concreteness and Fundamentals	.10

While these coefficients run somewhat higher, they are not enough so, on the whole, to warrant any but a tentative conclusion that probably the courses of study in public schools are more fully reflected in their work than is the case in others.

An interesting correlation of scorings of the courses of study themselves is:

General Excellence and Drill	.13
General Excellence and Concreteness	.94
Drill and Concreteness	.13

The high positive coefficient .94 is evidence that the excellences of concreteness stood out so clearly as to have had much more weight with the judges than did excellence in drill. This is confirmed by the comparative lack of relationship that drill is shown to have with any other rating.

Relationship of Course of Study and Time Expenditure

In a general survey one is impressed with the probability that the systems that have the best subject-matter in their courses of study have the smallest time cost. Evidence of such a condition is the omitting of much obsolete subject-matter, the appeal to interest, etc. How far this hypothesis is borne out is shown by the following correlations:

Table XXV

General Excellence with Time Cost

Without Home Work	−.13
Including Home Work	−.35

Excellence in Concreteness with Time Cost

Without Home Work	−.13
Including Home Work	−.35

Excellence in Drill with Time Cost

Without Home Work	.13
Including Home Work	.13

It should be carefully noted here that a *minus* coefficient indicates a *positive* answer in this table. A lack of correspondence in relative standing is the positive answer to the hypothesis stated above, i. e., high mark in course of study with low mark in time cost is what was expected and this is what is shown. This in connection with the fact that the coefficient of drill and time cost being + .13 is some indication that the systems whose courses of study show most provision for drill spend on the whole more time than those whose courses of study show high excellence in general and in concreteness.

Samples from Courses of Study

The best means of illustrating what the scorings of the courses of study determined and what the different types of courses ot study are, would be to print the courses of the entire twenty-six systems. Space forbidding that, the following selections are made. The work for grade III is chosen as the most helpful,

because of the growing custom of considering that, whatever the nature of the work of the first two grades, the work of grade III should be so planned as to facilitate definite progress in arithmetical knowledge.

Selections Illustrating General Excellence

From each of two systems ranking among the lowest five in course of study

3 B. Speer work. Simple work in addition and subtraction, following the plan in the Elementary Arithmetic.

3 A. Primary Book. First half page 26, second half page 41.

Grade III, Number

Exercises, mental and written, in addition, subtraction, multiplication and division of numbers.

The processes will be explained.

The multiplication table up to 12 will be made by the pupils and thoroughly committed to memory.

Drill in rapid addition.

Notation and numeration to five periods.

Table of weights, U. S. and English money. Problems in all tables learned.

Square and cubic measure. Troy and apothecaries' weights. Principles of multiplication.

From the system standing best in course of study

Grade III B

Scope: Review the work taught in preceding grades. (This review may require from four to six weeks.)

Addition and subtraction of numbers through twenty. Multiplication and division tables through 4's. Give much practice upon the addition of single columns. Abstract addition, two columns; the result of each column should not exceed twenty. The writing of numbers through one thousand. Roman notation through one hundred. Fractions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$. The object of the work of this grade is to make pupils ready in the use of the simple fundamental processes.

Book: Cook & Cropsey's New Elementary Arithmetic (for use of teacher), pages 1 to 46.

The chief difficulty in the work of this grade is in teaching the arithmetical forms as applied to concrete processes. Pupils should know very thoroughly the work given on pages 1 to 23, Cook & Cropsey's Arithmetic, before any new forms are taught. They have up to this time used the arithmetical signs and the sentence and have stated results only. New forms for addition and subtraction are first applied to concrete processes on page 24. No other forms should be taught until pupils are very

familiar with these. A drill should be given showing that these two forms are identical and that we must first know what we wish to use them for, if applied to problems. Write

$$\begin{array}{r} 9 \\ 2 \\ \hline \end{array}$$

upon the board and indicate your thought by the signs + and —,

$$\begin{array}{r} 9 \\ +2 \\ \hline 11 \end{array} \qquad \begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array} \qquad \begin{array}{l} 9 \text{ apples} \\ +2 \\ \hline 11 \text{ apples} \end{array} \qquad \begin{array}{l} 9 \text{ apples} \\ -2 \\ \hline 7 \text{ apples} \end{array}$$

Pupils should be very familiar with these forms before any written concrete work is given.

When the new form for multiplication is introduced this drill should be repeated:

$$\begin{array}{r} 9 \\ +2 \\ \hline 11 \end{array} \qquad \begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array} \qquad \begin{array}{r} 9 \\ \times 2 \\ \hline 18 \end{array}$$

Nothing new should be added to this until pupils can use these forms without confusion.

When presenting the new forms for division and partition the same method may be used, but pupils should use the form for division some weeks before using the same form for partition. It is not necessary to use the division form for partition until the last four weeks of the term, and not even then, if there seems to be any danger of confusion in using the same form for both processes. The terms division and partition should not be used. The terms *measure*, and *finding one of the equal parts* can be easily understood. Pupils should be able to read arithmetical forms well, before any use is made of these forms in their application to written concrete work.

All concrete problems should be simple and within the child's experience.

Grade III A

Scope: 1. Review the work of Grade 3B.

2. Abstract addition of three columns. Subtraction, using abstract numbers through thousands. Addition and subtraction of United States money. Multiplication and division tables through 6's. Multiplication and division of abstract numbers through thousands, using 2, 3, 4 and 5 as divisors. Addition and subtraction by "endings" through 2+9 last term of month. Writing numbers through ten thousands. Roman notation through one hundred. Fractions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$.

3. Application of fundamental processes to simple concrete problems, of one step.

4. Measures used—*inch, foot, yard, square-inch; pint, quart, gallon; peck, bushel; second, minute, hour, day, week, month, year.* Use actual measures.

Books: (In hands of pupils) Walsh's New Primary Arithmetic, pages 1 to 68.

(For teachers' use) Cook & Cropsey's New Elementary Arithmetic, pages 46 to 85, Article 105.

Even with only the work of a single grade to judge from, one has no difficulty in recognizing the wide difference in the excellence of these courses. As may be seen from table XXVIII, page 73, in the rating they stand about thirty steps apart, i. e., the one from which the third illustration was taken has a score of 65 while the others have scores of 32 and 39 respectively.

Selections Illustrating Excellence in Drill and in Concreteness

From the system ranking next to the best in drill.

Grade III B

I. OBJECTIVE.

1. *Work.*
   - (a) Fractions. Review previous work. Teach new fractions; 7ths, 10ths, and 12ths.
   - (b) Notation, numeration, addition and subtraction of numbers to 1000.
   - (c) Liquid and dry measures.
   - (d) United States money.
   - (e) Weights.
2. *Objects and Devices.*
   - (a) Counting frame.
   - (b) Splints, discs for fractions, etc.
   - (c) Shelves.
   - (d) Liquid and dry measure.
   - (e) United States money.
   - (f) Scales.

II ABSTRACT.

1. *Work.*
   - (a) Counting to 100 by 2's, 10's, 3's, 4's, 9's, 11's, 5's, beginning with any number under 10; counting backwards by same numbers, beginning with any number under 100.
   - (b) Multiplication tables. Review tables already studied. Teach 7 and 9.
   - (c) Drill in recognizing sum of three numbers at a glance; review combinations already learned; 20 new ones.

II. ABSTRACT—*Continued.*

2. *Devices.*
   (a) Combination cards, large and small.
   (b) Wheels.
   (c) Chart for addition and subtraction.
   (d) Fraction chart.
   (e) Miscellaneous drill cards.
   (f) Pack of "three" combination cards.

PRINCE'S ARITHMETIC, Bk. III, Sects. I & II.

SPEER'S ELEMENTARY ARITHMETIC, pp. 1–55.

Shelves: See II A.

Combination Cards—large and small. These cards should contain all the facts of multiplication tables 3, 6, 8, 7, and 9. As:—

7 x 1	2 x 7	7 ÷ 1	21 ÷ 3
1 x 7	7 x 3	14 ÷ 2	21 ÷ 7 etc.
7 x 2	3 x 7	14 ÷ 7	

For use of these cards, see directions in I B.

Wheels for Multiplication and Division:
See directions under II A.

Chart for Adding and Subtracting:
For directions see II B and II A.

Add and subtract 2's, 3's, 4's, 5's, 9's, 10's, 11's, 12's, 15's, and 20's.

Fraction Chart shows, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$

Miscellaneous Drill Cards:
For directions see I A.

"Three" Combination Cards:
For use see I A.

Grade III A.

I. OBJECTIVE.

1. *Work.*
   (a) Fractions previously assigned.
   (b) Notation, numeration, addition, subtraction, multiplication, and division of numbers to 1000.
   (c) Long and square measures.
   (d) Weights.
2. *Objects and Devices.*
   (a) Counting frame.
   (b) Splints, discs for fractions, etc.
   (c) Shelves.
   (d) Scales.

I. ABSTRACT.

1. *Work.*
   (a) Counting to 100 by any number from 2 to 12 inclusive, beginning with any number under 10; counting by same numbers backward, beginning with any number under 100.

I. ABSTRACT—*Continued.*

1. *Work—Continued.*
   - (b) Multiplication tables—all tables.
   - (c) Drill in recognizing sum of three numbers at a glance; review combinations already learned; 20 new ones.
2. *Devices.*
   - (a) Combination cards—large and small.
   - (b) Wheels.
   - (c) Chart for adding and subtracting.
   - (d) Chart for fractions.
   - (e) Miscellaneous drill cards.
   - (f) Pack of "three" combination cards.

PRINCE'S ARITHMETIC, Bk. III, sects. III to VI inclusive.

SPEER'S ELEMENTARY ARITHMETIC, pp. 56–104.

Shelves: See II A.

Combination Cards: large and small. The cards should contain all the facts of the multiplication tables 11 and 12, also the most difficult combinations from the other multiplication tables. As:—

12 x 1	12 ÷ 1	24 ÷ 2
1 x 12	12 ÷ 12	24 ÷ 12 etc.
12 x 2	12 ÷ 2	
2 x 12	12 ÷ 3	

For use of cards, see directions in I B.

Wheels for Multiplication and Division:

See directions under II A.

Chart for Adding and Subtracting:

For directions see II B and II A.

Add and subtract 6's, 7's, 8's, 13's, 14's, 16's, 17's, 18's, and 19's. Review other numbers under 20.

Chart for Fractions shows all fractions already assigned.

Miscellaneous Drill Cards:

For directions see I A.

From the system ranking best in concreteness.

Mathematics.—If the children are actually doing work which has social value, they must gain accurate knowledge of the activities in which they are engaged. They will keep a record of all expenses for materials used in the school, and will do simple bookkeeping in connection with the store which has charge of this material. In cooking, weights and measures will be learned. The children will also keep accounts of the cost of ingredients. Proportions will be worked out in the cooking recipes. When the children dramatize the life of the trader, in connection with history, they have opportunity to use all standards of measurements. Number is demanded in almost all experimental science work; for instance, the amount of water contained in the different kinds of fruit, or the amount of water evaporated from fruits under different conditions

(in drying fruits). All plans for woodwork will be worked to a scale and demand use of fractions. When the children have encountered many problems which they must solve in order to proceed with their work, they are ready to be drilled on the processes involved until they gain facility in the use of these. The children should be able to think through the problems which arise in their daily work, and have automatic use of easy numbers, addition, subtraction, multiplication, short division and easy fractions.

As one reads these two samples of excellence he must find that each is so excellent in its one strong feature that it is not good; that work according to either must suffer; that what each needs is what the other has. Such a synthesis is represented in the next illustration.

A Combination of Excellences

September. 1. Measure height, determine weight. From records determine growth since Sept., 1905. 2. Learn to read thermometer. Make accurately, scale one-fourth inch representing two degrees on paper one inch broad. Find average temperature of different days of month. Practice making figures from 1 to 100 for the thermometer scale. Count 100 by 2's. Develop table of 2's. 3. Make temperature chart. 4. Measure and space calendar, making figures of size appropriate to inch squares. Learn names of numbers to 30. 5. Make inch wide tape measure for use in nature study, number book and cubic inch seed boxes. 6. Review telling time. A. In addition to above; analyze numbers from 11 to 40 into tens and ones. Walsh's Primary Arithmetic to top of page 10.

October. Problems on calendar,—number of clear, of cloudy and of rainy days in September. Compare with September 1905, 1904, 1903, 1902; temperature chart and thermometer; height and weight. Lay off beds for tree seeds, plant the same. Make envelopes for report cards. Drill on combinations in the above. Make rod-strings and hundred foot strings for determining distance wing seeds are carried from plants. Practice making figures from 1 to 100 for thermometer scale. Develop table of tens. A. In addition to the above analyze numbers from 40–50; into tens and ones. Primary Arithmetic, pages 10–22. Teach pupils to *add at sight*.

November.—From wall calendar count number of clear days, of cloudy days and of rainy days in October. Compare with September; with October of 1905, of 1906. Find average daily temperature; 8:30 A.M., 1.00 P.M. What kind of trees grow fastest. Measure growth of twigs of different kinds of trees. Compare this year's growth with that of last year and of year before last. Compare rate of growth of different kinds of trees, as oak, willow, Carolina poplar and elm. Develop table of 5's

from lesson with clock dial; review 2's and 10's. Practice making figures from 1 to 100 for the thermometer scale. Learn words representing numbers as well as figures. Make seed envelope. A. Analyze numbers from 60–65 into tens and ones. Primary Arithmetic.—B- pages 17–26, A- pages 39–49.

Last six weeks of first term.—Continue finding average daily temperature. From wall calendar count number of clear, of cloudy and of rainy days in November. Compare with November 1906, 1905. Continue measurements on growth of trees. Drill on telling time from clock dial. Practice making figures from 1 to 100 for thermometer scale. Continue learning words representing numbers. Review tables of 2's, 5's, 10's; learn table of 3's. Primary Arithmetic.—B-pages 27–40. Analyze numbers from 11–30 into tens and ones. Primary Arithmetic, A-pages 49–61. Analyze numbers from 66 to 100 into tens and ones. In January review all facts in number book. Drill on tables.

(Only the first *one-half* of the third year's course shown).

The system from which this last selection is taken had the following remarkable rankings: 3d best in general excellence, 2d best in concreteness, and 5th best in drill. And as measured by the tests of this study, this system stood 4th from the best in abilities and spent a little less than the medium amount of time.

The Course of Study Situation

The reader who believes in the potency of the course of study could hardly have been satisfied with the above findings. So much of what has been shown is negative that one is tempted to doubt the validity of the research in his mental revolt against what *a priori* seems an impossible conclusion; but on second thought one sees that while the findings are negative they are a negation of the potency of the course of study in present practice as represented by the group of systems studied and not necessarily a negation of the potency of the course of study as it is probably functioning in certain individual systems and as it may function in all. The situation seems to be that the course of study is not at present the factor that it ought to be in producing abilities. In certain systems it is evidently working well, but in others there is a wide disparagement between excellence in abilities and excellence in course of study. Just how much the systems diverge is best illustrated by the next table.

TABLE XXXVI

Serial Standing in Abilities and in Course of Study.

SYSTEMS	Course of study[1]	Abilities		
		Average[2]	Reasoning	Fundamentals
I	1	11	14	9
V	2	25	26	21
II	3	13	16	11
X	4	10	15	6
IV	5	12	4	20
VII	6	23	21	25
VIII	6	6	12	5
XVI	8	16	7	23
XXVI	9	23	24	22
III	9	8	11	8
XIII	11	17	18	13
XVIII	12	18	9	24
XIV	13	20	19	19
XXIV	14	9	2	18
XVII	14	5	3	12
XV	16	6	10	7
XXIII	16	1	1	1
XX	18	4	8	3
VI	19	13	13	14
IX	19	19	20	16
XIX	21	26	25	26
XXI	22	13	17	10
XXV	22	2	4	2
XII	24	22	22	17
XXII	25	3	6	4
XI	26	20	23	15

This table shows only *two* systems whose standings in course of study and abilities correspond exactly; only *four* others where the correspondence is fairly close; *eleven* where the abilities are decidedly better and *nine* where the course is decidedly better.

The best single measurement of this disparagement is the coefficients of correlation which as given on page 74, are —.16 in reasoning with course of study and —.09 in fundamentals with course of study.

It is evident that just as was shown in Part III that in present practice the amount of time cost is no criterion of abilities, so

[1] This column is the serial standing in general excellence of course of study. Compare table XXVIII, p. 73.

[2] This column is the average of the serial standings in reasoning and fundamentals, the separate serial standings of which are found in the next two columns. Compare table XXII, p. 55

these findings indicate that a good course of study is no guarantee of good abilities. This is another emphatic expression of the diversity of practice in present day education. The nine systems whose courses are better than their abilities are evidently relying too exclusively on the efficacy of the curriculum, while the eleven systems whose courses are not so good as their abilities are evidently not making full use of the course of study as a means of producing abilities. In other words, just as in Time Expenditure it was found that certain systems are relying too exclusively on merely *spending time* on arithmetic, so here it is evident that certain systems are relying too much on merely having a good course of study; and just as a few other systems are probably not spending quite enough time, so here it is shown that the work of some systems would probably be better if their courses of study were better.

That the better courses of study even in present practice, do tend to go with the better abilities is convincingly shown by computing coefficients of correlation between course of study excellence and abilities, using the thirteen systems ranking less than the median in course of study excellence as one group and using those ranking better than the median as another group. The coefficients are:

Course of Study less than median
with reasoning —.13
with fundamentals .13

Course of Study more than median
with reasoning .56
with fundamentals .13

These coefficients not only indicate that the systems with better courses of study tend also to have adequate time allotments, good supervision, etc., but they also point to another significant fact, viz., that ability in fundamentals is much less closely related to excellence in course of study than is reasoning. The comparatively large coefficient of +.56 for reasoning with the best thirteen courses of study will be gratifying to those who believe that ability to think is amenable to the influence of the kind of subject matter studied.

As to improvements in courses the author's best suggestion is to point to the types of excellence illustrated above. The third course illustrated on page 80 is the best of the twenty-six in all particulars except concreteness, but the author believes so heartily in the value of having pupils deal with subject-matter that is most worth while from the broad educational standpoint as well as from the standpoint of arithmetic that he prefers the course from which the last illustration was taken. With those who still believe in drill *per se* the author cannot agree but this investigation is conclusive evidence to him that adequate provision for drill is one essential to the best production of abilities. What a majority of the *better* courses of the twenty-six used in this research most need is a better provision for *drill.* The best course of study will embody the good points of the two illustrations given on pages 82 to 85. Such a synthesis is well illustrated in the course from which the illustration is given on page 85.[1]

Stated in another way, future improvement in courses of study seems to the author to lie in the direction of indicating the place of drill in the educative process. The best courses of study will continue to be made in accord with the *social aim* and *functional psychology;* the next step in improvement is to so arrange the subject-matter as to indicate how it may be so taught as to afford sufficient discipline of the right kind.

GENERAL CONCLUSIONS

1. The first general conclusion with regard to this study concerns the possibilities of such work. It does not seem too much to conclude that the general method of this research is a means by which hypotheses are to be tested and opinions to become facts. The author forms this conclusion even more because of the willingness with which the scores of helpers have cooperated than because of the method—valuable as it is.

[1] Two other noteworthy courses of study that were unfortunately written too late to be utilized in this investigation are those of Western Illinois State Normal School (Macomb, Illinois) and San Francisco. The former was worked out under the direction of Prof. Bonser of the Macomb Normal, and for the formal aspects of the first five grades of the latter Prof. Suzzallo, now of Teachers College, Columbia University, is largely responsible.

2. Probably the truest single expression of the findings of this study is summed up in the one word, *diversity*. For many students of American tendencies the best word would probably be *chaos*. Freedom and initiative are here seen to have led educational practice in widely varying paths. Certain paths are those of legitimate differentiation, but others are *waste*. Striking evidence of this is: (1) A variability of scores among systems of 356 to 914 points, with an average deviation of 112 in reasoning; and 1,841 to 4,099, with an average deviation of 421 in fundamentals; (2) A variability of mistakes among systems of 14.5% to 4.7% of all the steps attempted in addition, and 45.1% to 14.4% of all the problems attempted in reasoning; (3) A variability of 507 to 1,854 week-minutes with an average deviation of 222 week-minutes in time expenditure, without-home study; and 507 to 2,179 week-minutes with an average deviation of 269 week-minutes, including home study; (4) A variability in average ratio of time to abilities of from 2.26 to .64; (5) A difference in course of study excellence which can hardly be put in words. Samples are given on page 80.

These wide variabilities are the more striking when one recalls that the study concerns only the work of the first six grades, in which, by practically common consent, the work needs to be fairly uniform in product if not in process.

3. The greatest need shown by the research is standards of achievement. That the great variability herein shown would exist if school authorities possessed adequate means of measuring products is inconceivable; and it is believed that the present study will help standardize the work in arithmetic for the first six grades. Anyone who wishes may know how his system or school compares with the representative systems of the country. Relative standings may be determined as follows:[1]

In abilities by (1) giving the tests of this study (see pages 10 to 12), according to the conditions stated on page 13, and the directions to pupils on page 14, (2) scoring the papers according to the method given on pages 15 to 19; (3) comparing the scores in reasoning with those in table III, in fundamentals with those in table IV, in accuracy with those of tables VIII and IX.

[1]It will be recognized that no special knowledge of statistical method is necessary for these applications of standards. For the benefit of those who may wish to continue the work on correlations, complete data are given in the various tables.

In time expenditure by (1) computing time cost in week-minutes;[1] (2) comparing this cost with those of table XXI; (3) computing ratios of time cost to scores, as was done for table XXVII; (4) comparing ratios with those of table XXVII.

Course of study excellence may be inferred by comparing the course for Grade III with the samples given on pages 80 to 86.

4. The author believes that the systems studied are sufficiently representative, the data sufficiently reliable, and the method sufficiently accurate and thorough to warrant the hope that the conclusions previously stated as summaries of each of the last three parts of the study will commend themselves as guides to future progress. The conclusions as to the complexity of the nature of the products of arithmetic work are the summary of Part II (page 42); those as to the lack of relation between time expenditure and abilities are found, together with suggestions for determining a rational time allotment, at the close of Part III (page 69); those concerning the present status and future improvement of courses of study complete Part IV (page 86).

5. Doubtless the most helpful generalization possible from this study is that there is no *one* factor that produces abilities, there is no single *summum bonum* in teaching arithmetic. The course of study may be the most important single factor, but it does not produce abilities unless taught. The other essential conditions for successful teaching are children and teachers of usual abilities, a reasonable time allotment, intelligent supervision, and adequate measuring of results by tests.

6. As to the individual systems, a general conclusion that purports to be more than an estimate would be presumptuous with the present limited knowledge on the part of the author. The following table gives the relative position of each system in each phase of the study with the author's estimate of its status and suggestions for future improvement. A closer study of each of these systems for the purpose of making more helpful and more reliable suggestions would be a pleasurable means of repaying the debt which the author cheerfully acknowledges; but time and space limitations do not at present permit.

[1] For the explanation of *week minutes* see note p. 48, and sample reading of table XXI p. 52.

TABLE

Relative standings in abilities and in factors producing them. Rankings

Systems	Abilities			Time		Ratio of time to abilities			Accuracy		Course of study			Supervision[2]		
	Average	Reasoning	Fundamentals	Without Home Study	Including Home Study	Reasoning	Fundamentals	Average	Reasoning	Fundamentals[1]	General Excellence	Drill	Concreteness	Supervisor or Superintendent	Principal	Tests
XXIII	1	1	1	14	8	2	1	2	3	15	16	9	21	y	y	?
XXV	3	4	2	2	6	17	15	19	16	14	22	21	23	y	y	?
XXII	4.5	5	4	1	1	22	23	24	8	1	25	4	26	y	y	n
XX	5	7	3	15	22	7	2	5	18	2	18	2	24	y	y	?
XVII	7.5	3	12	21	16	3	7	3	2	4	14	18	13	n	n	y
VIII	8	11	5	18	24	8	5	8	13	16	6	18	8	n	n–	n
XV	8	9	7	16	10	11	8	11	6	11	16	6	20	n	y	n–
III	9	10	8	6	2	16	16	17	4	19	9	24	15	y	y	y
XXIV	10	2	18	7	3	10	21	13	10	22	14	14	6	y	y	y
X	10	14	6	5	5	18	14	20	9	7	3	14	4	y	n	y
I	11	13	9	9	12	15	10	16	5	3	1	1	1	y	n	y
IV	12	4	20	26	25	1	4	1	17	13	5	11	1	y–	n	y–
II	13	15	11	17	13	14	7	14	23	18	3	16	6	y	n	y
XXI	13	16	10	4	9	21	19	23	25	25	22	11	22	n	y	y
VI	13	12	14	11	7	13	11	15	7	6	19	10	17	n	n	n
XVI	14.5	6	23	12	18	6	18	10	1	17	8	8	4	y	y	y
XIII	15	17	13	25	23	4	3	4	11	10	11	17	13	n	y	n
XVIII	16	8	24	19	15	5	13	6	14	5	12	18	10	n	y	y
IX	17.5	19	16	22	26	9	5	9	22	8	19	25	15	y–	y	y–
XI	18.5	22	15	10	11	20	12	21	26	26	26	26	17	y	y	y
XIV	18.5	18	19	23	19	7	6	7	20	21	13	13	12	y	y	n
XII	19	21	17	13	17	17	13	18	21	20	24	22	25	y	n	y
XXVI	22.5	23	22	3	4	24	22	26	12	23	9	5	9	y	y	y
VII	22.5	20	25	24	20	12	9	12	19	12	6	2	11	y–	y	n–
V	23	25	21	8	14	23	20	24	24	9	2	7	3	y–	y	n
XIX	25	24	26	20	21	19	17	22	15	24	21	22	17	y	y	y

[1] Percentages of mistakes in addition were used to measure accuracy in fundamentals here as previously, cf table IX, p. 27.

XXXVII

are in serial order, 1 = lowest, 2 = next, etc.; y = yes; n = no.

AUTHOR'S ESTIMATE OF STATUS AND SUGGESTIONS FOR FUTURE IMPROVEMENT

More accurate in formal work than in reasoning. Drill and testing probably needed.

Accuracy fair. Pupils evidently slow. Course of study good. Drill and testing probably needed.

Ratio of time to abilities excellent. Course of study very low in drill and highest in concreteness. Testing and probably more time needed.

Much more accurate in reasoning than in formal work. Pupils seemed slow. Course of study shows very little drill but is very good in concreteness.

Pupils varied widely. Time rather high. Supervision needed.

Drill element better than remainder of course of study. Supervision needed.

Course of study much better in concreteness than in drill. Drill and official tests needed.

Possibly too little time. Increased accuracy in reasoning would help.

Course of study lacks concreteness. Possibly too little time to make reasoning as good as formal work.

Ratios of time to abilities good but possibly too little time. Course of study *as printed* lacks concreteness.

Course of study lacks in helpfulness to teacher. More supervision would probably help increase accuracy and rapidity.

Pupils slow but more accurate in reasoning than in formal work. Excessive time cost. Supervision needed.

Pupils did very well in accuracy of reasoning. Course of study lacks in general helpfulness except drill.

Pupils worked very deliberately. Few mistakes. Drill lacking in course of study.

Supervision and tests would help accuracy. Conditions otherwise good.

Great inaccuracy in reasoning. Formal work much better. Course of study lacks concreteness.

Time high. Supervision and tests would help.

Better in *amount* of formal work than reasoning, but better in accuracy of reasoning.

Much more accurate in reasoning than in formal work. Course of study lacks concreteness. Time high.

Excellent conditions. *Few* errors. Course could be more concrete.

Very good conditions except time cost too high and course of study might be better.

Conditions uniformly good. Reasoning a little better than formal work. Course of study excellent.

Excellent conditions except course as printed. Increased accuracy in reasoning would help.

Course not so good as work. Time too high.

Course not so good as work. Otherwise good conditions.

Very good conditions except accuracy in reasoning.

[2] A minus sign indicates qualified answer. For blank by which data was gathered for all public schools, see p. 94.

APPENDIX

TEACHERS COLLEGE
COLUMBIA UNIVERSITY
NEW YORK

As the supervision of the arithmetic work of the pupils whom I tested is a factor on which my study ought to offer some help, may I depend on you to answer the following questions at your early convenience?

Realizing that you may not be able to give *exact* answers to all the following, I suggest that if you are in doubt as to the accuracy of an answer you place one of the following letters after each doubtful answer: A, if nearly certain; B, if less certain; C, if pure guess. But please *answer all.*

I. As the pupils whom I tested passed through the respective grades, did the arithmetic work of their teachers have supervision by superintendent or other supervisor, other than that given by the principal?

If so, approximately how many teachers were in charge of each supervisor?

In what grade did the arithmetic work receive this special supervision?

Approximately what portion of each superintendent's or supervisor's time was given to arithmetic?

For what other subjects was each supervisor primarily responsible?

II. Did your principals supervise instruction in arithmetic?

If so, approximately how many teachers were thus supervised by each principal?

In what grades did the principal supervise the arithmetic work?

For what other subjects was the principal responsible?

III. Did you make use of official tests as a means of supervising the work of the pupils I tested?

If so, in what grades?

By whom were questions made out?

How often were tests given?

What purposes did these tests serve? To determine fitness for promotion, to check up the work of the teachers, or some other purpose?

I trust I need not say that no mention will be made of this data in connection with the *name of your system.*

Answers to all questions will help most—even if you think best to mark them with *C.*

Sincerely,

Dr. Rice's Study of Arithmetic

So far as the author is aware, the only previous comprehensive attempt to determine and account for arithmetical abilities is that of Dr. Rice.[1] While, as will be pointed out, there are several limitations to this study, its importance can hardly be overestimated. Previous to it, practice was almost entirely based on opinion; and the success of practice was almost entirely judged by the enthusiasm of those who defended their opinions.

After scoring the test papers of some six thousand children from seventeen schools of seven cities, Dr. Rice discusses twelve possible factors of successful work. These with a brief summary of his conclusion as to each are as follows:[2] (1) *Home environment.*—"As in spelling, so in arithmetic, this mountain, on close inspection, dwindles down to the size of a molehill;" (2) *Size of classes.*—" No allowance whatever is to be made for the size of the class in judging the results of my test;" (3) *Age of pupils.*—" This factor can be held accountable for the difference shown to only a slight degree if at all"; (4) *Time of day.*—"This likewise is not a determining factor;" (5) *The time devoted to arithmetic in the school.*—"A glance at the figures will tell us at once that there is no direct relation between time and result;" (6) *The amount of home work required.*—This fact is also de-

[1] *Forum*, Vol. 34, p. 281 and p. 437.
[2] See Thorndike's *Educational Psychology* for a more comprehensive summary.

nied to be a controlling one as the highest five schools had practically abandoned home work, while the lowest ranking city required most; (7) *Method of teaching.*—In the schools that passed the tests satisfactorily no special method had been in use; (8) *Teaching ability.*—" Teachers of most successful schools had no better ability than those of unsuccessful schools;" (9) *Course of study.*—By a curious line of argument Dr. Rice reaches the conclusion that the course of study is not a factor to be considered because the tests were fair to pupils who had been taught by all courses of study; (10) *Superintendent's training of teachers.*—This was as much in vogue in localities that did poorly as in those that did well; (11) and (12) *Superintendents' establishing of standards and testing for results.*—These Dr. Rice concludes to be the large and controlling factors.

Improvements Attempted in the Present Study

Any subsequent study should take account of and profit by those extant. Hence there is no spirit of adverse criticism of Dr. Rice's work in the following enumeration of attempted improvements. These attempts may be grouped in two classes,—(1) those that pertain to the data used and (2) those that pertain to the method of securing and handling data.

The chief improvement in the data used constitutes Part IV of this study. It is an attempt to take account of the influence of the kind and arrangement of subject-matter, the place of drill, etc., in the Courses of Study. As *what shall constitute the Course of Study* has been one of the chief battle-grounds of the "new" vs. the "old education," it is difficult to see why Dr. Rice treats it so inadequately.

The improvements attempted in the gathering and handling of data are chiefly those of refinement, and they could hardly have been planned for without the benefit of Dr. Rice's and other pioneer studies. The objections to Dr. Rice's procedure will be stated dogmatically here for the sake of brevity. The full statement of the improvements that were brought about appears in Part I.

1. Dr. Rice's tests attempt to measure ability in both reasoning and fundamentals. As was shown in Part II, these are different abilities and should be so measured.

2. Dr. Rice's tests contain subject-matter that some of the pupils tested may not have covered. This evidently makes his tests—to this extent—a test of what the teachers have taken their pupils over rather than a test of what power the pupils have.

3. There is no statement made as to time limit. The reader does not know whether all pupils were given the same amount of time or whether all were encouraged to take time enough to do their best, or whether there was variation in this respect.

4. We do not know how the tests were placed before the pupils. Dr. Rice says, "in each instance during my presence;" and the inference is that they were not given by himself, but whether they were given by the principal, superintendent, or teacher is not shown. Any one who has observed the varying effects of different personalities—to say nothing of varying directions—on pupils, will realize the absolute futility of relying on results obtained with this factor varying.

5. So far as can be determined, the method of scoring is not the best. Dr. Rice may have meant to arrange his problems in order of difficulty, but unless their arrangement is based on preliminary tests there is little certainty that the order is correct. And certainly the more difficult ought to weigh more in the score than the easier.

6. Too little of the data and the computations are made available. Every quantitative study ought to afford opportunity for (1) recomputing, using the data as presented; (2) computing other commensurate data for purposes of confirming or refuting the conclusions. More than averages should be printed. At least some measure of the variability should be given if not entire distributions.

7. The method of showing relations between achievement and the various factors could be much shortened, made more concise and vastly increased in accuracy by stating relations in terms of co-efficients of correlation.[1]

[1] cf. Thorndike's *Educational Psychology*, p. 76.

TABLE XXXVIII

Scores of the twenty-six systems in Reasoning, according to different weightings

Graduated as in last column of Table II, page 18		Graduated as in next to last column of Table II, page 18		Counting each problem 1		Counting only first six prob., weighting as in last col. of Table II	
System	Scores	System	Scores	System	Scores	System	Scores
XXIII	356	XXIII	379	XXIII	341	XXIII	342
XXIV	429	XXIV	459	XXIV	410	XVII	389
XVII	444	XVII	498	XVII	411	XVI	389
IV	464	XVI	504	XVI	437	XXIV	396
XXV	464	XXV	508	IV	438	IV	420
XXII	468	IV	511	XXII	438	XXII	423
XVI	469	XXII	520	XXV	442	XX	426
XX	491	XX	555	XX	456	XXV	438
XVIII	509	XVIII	573	XVIII	473	III	445
XV	532	III	604	III	487	XVIII	452
III	533	VIII	609	XV	490	VI	455
VIII	538	XV	612	VIII	497	I	466
VI	550	I	640	VI	500	VIII	468
I	552	VI	644	I	507	XIII	497
X	601	X	691	X	549	X	502
II	615	XIII	717	II	559	IX	503
XXI	627	XXI	734	XXI	568	XV	508
XIII	636	II	741	XIII	569	XIV	514
XIV	661	XIV	742	XIV	590	II	516
IX	691	XII	888	VII	630	XXI	532
VII	734	IX	897	IX	633	XII	536
XII	736	VII	916	XII	646	V	549
XI	759	XI	962	XI	664	XIX	564
XXVI	791	XXVI	1002	XXVI	688	XXVI	569
XIX	848	XIX	1145	XIX	710	XI	576
V	914	V	1266	V	748	VII	661

SPECIAL TEST IN REASONING

[Given to measure the reliability of the reasoning test used in this study. See pages 100, 101.]

Solve as many of the following problems as you have time for; work them in order as numbered:

1. There were 37 pupils in a certain *sixth* grade; 22 were promoted into the *seventh* grade, and 17 were promoted into the *sixth* grade from the *fifth* grade. How many were then in the *sixth* grade?

2. A man whose salary is \$20 a week spends \$14 a week. In how many weeks can he save \$300?

3. A school paid \$103 for desks and one chair. The chair cost \$4, and the desks \$3 each. How many desks were purchased?

4. A train has just passed the second station of its trip of 95 miles. The first station is 25 miles from the start; and the second station is 17 miles from the first. How many miles of the trip are left?

5. If the retailer pays \$2.75 per box of 120 oranges and sells the oranges at 35¢ a dozen, what does he make on a box?

6. If the retailer buys larger oranges, 96 to a box, at the same price and sells them at 50¢ per dozen, how much does he make per box?

7. A man gave away \$2,200 as follows: $\frac{1}{2}$ to his son, $\frac{1}{4}$ to his daughter, $\frac{1}{8}$ to his brother, and the remainder to a friend. How much did the friend receive?

8. A dealer paid \$30 for a dozen pairs of shoes; he sold them at a gain of \$.75 a pair. What would you have to pay for a pair?

9. Bought a pony and cart for \$200. Sold the pony for \$150, gaining \$25. What did the cart cost?

10. In buying 48 cans of tomatoes how much is gained by buying two cases of 2 dozens each @ \$2.90 a case, over buying at the rate of 3 cans for 45 cents?

11. A man spent $\frac{2}{3}$ of his money and had \$18 left. How much had he at first?

12. The two largest states in the union are Texas and California. Texas exceeds California in area by 106,000 square miles. The sum of their area is 418,000 square miles. Find the area of each.

THE RELIABILITY OF THE REASONING TEST

As a Measure of Systems

The reliability of the reasoning test used as a measure of the abilities of systems was measured by:

(1) Giving the test used in this study to two groups of fifty pupils each, chosen at random. These groups made combined scores of 342 and 323 respectively, counting a score of *one* for each problem reasoned correctly.

(2) Giving the test[1] on pages 98 and 99 to the same pupils, after an interval of about three weeks. The groups made combined scores of 344 and 304 respectively.

(3) Comparing the ratios of the combined standings in the two tests. The ratio for the first group is $\frac{342}{344}$ or .99 and for the second group $\frac{323}{304}$ or 1.06.

The very slight difference between these ratios shows that there is only a small variation in the measures secured for groups of fifty pupils when measured by different tests. As the measures used in this study are the combined scores of one hundred pupils,[2] they are probably more reliable than the above indicates.

As a Measure of Individual Pupils

The reliability of the reasoning test as a measure of the ability of individual pupils was measured by computing the coefficient of correlation using the individual scores of the two tests. The Pearson coefficient was found to be .57 with a P. E. of .045, *i. e.*, the chances are *one* to *one* that if an infinite number of pupils were tested the true correlation would not be more than .62 or less than .52.

[1] While this test is made up of problems entirely different from those of the test used in the study, it was just as carefully prepared and is meant to embody the same conditions. See page 10.

[2] For a few of the smaller systems the measure was necessarily based on less than 100 pupils.

These findings show that the combined scores of groups of pupils of fifty or more vary only comparatively little when measured by different tests; the individual scores of pupils tend to vary to the extent that the correlation is only about .57 instead of 1.00 as it would be if they did equally well in all tests at all times. The reason for this agreement of group standings and disagreement of pupil standings is that while individual pupils vary in scores made, the variabilities within a group of fifty or more pupils are such as to practically neutralize each other.

To accompany top of page 17—to show more in detail the method of scoring the fundamentals.

Work as many of these problems as you have time for; work them in order as numbered.

1. Add 2375 *All columns correct=score of 4 in addition.*
4052 *3 columns correct=score of 3 in addition, etc.*
6354
260
5041
1543

2. Multiply 3265 by 20. *Multiplying by 0 and by 2 correctly=score of 2 in multiplication.*
3. Divide 3328 by 64. *If all correct=score of 1 in subtraction, 2 in multiplication, 2 in division.*
4. Add 596
428
94
75
302 *3 addition.*
645
984
897

5. Multiply 768 by 604. *2 addition, 3 multiplication.*
6. Divide 1918962 by 543. *3 subtraction, 4 multiplication, 4 division.*
7. Add 4695
872
7948
6786
567 *4 addition.*
858
9447
7499

8. Multiply 976 by 87. *4 addition, 2 multiplication.*
9. Divide 2782542 by 679. *2 subtraction, 4 muliplication, 4 division.*
10. Multiply 5489 by 9876. *7 addition 4 multiplication.*
11. Divide 5099941 by 749. *2 subtraction, 4 multiplication, 4 division.*
12. Multiply 876 by 79. *3 addition, 2 multiplication.*
13. Divide 62693256 by 859. *4 subtraction, 5 multiplication, 5 division.*
14. Multiply 96879 by 896. *7 addition, 3 multiplication.*

N. B. It should be noted that scores of systems of schools are given *per* 100 *pupils* on PAGE 21 and throughout the study.